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Small surfaces of Willmore type in Riemannian manifolds. (English) Zbl 1202.53056

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Summary: We investigate the properties of small surfaces of Willmore type in three-dimensional Riemannian manifolds. By small surfaces, we mean topological spheres contained in a geodesic ball of small enough radius. In particular, we show that if there exist such surfaces with positive mean curvature in the geodesic ball $B_r(p)$ for arbitrarily small radius r around a point p in the Riemannian manifold, then the scalar curvature must have a critical point at p. As a byproduct of our estimates, we obtain a strengthened version of the non-existence result of Mondino (to appear) that implies the non-existence of certain critical points of the Willmore functional in regions where the scalar curvature is non-zero.

MSC:

53C40 Global submanifolds

Keywords:

topological spheres; geodesic ball; Willmore functional; scalar curvature

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