Oskolkov, K. I.; Chakhkiev, M. A.

On Riemann "nondifferentiable" function and Schrödinger equation. (English. Russian original)

Zbl 1207.26010

Proc. Steklov Inst. Math. 269, 186-196 (2010); translation from Trudy Mat. Inst. Steklova 269, 193-203 (2010).

For $\{x,t\} \in \mathbb{R}^2$, the authors consider the function

$$\psi(x,t) = \sum_{n\in\mathbb{Z}\setminus\{0\}} \frac{e^{\pi i(tn^2+2xn)}}{\pi i n^2},$$

which is a generalization of Riemann's nowhere-differentiable function, and also it is a generalized solution of the Cauchy initial value problem for the Schrödinger equation. The authors study some of the partial derivatives of $\psi(x,t)$, as far as they show that the local Lipschitz-Hölder smoothness exponent of it in the variable t equals $3/4$ almost everywhere on $\mathbb{R}^2$.

Reviewer: Mehdi Hassani (Zanjan)

MSC:

26A27 Nondifferentiability (nondifferentiable functions, points of nondifferentiability), discontinuous derivatives

35Q41 Time-dependent Schrödinger equations and Dirac equations

Cited in **1** Document

Keywords:

Riemann's nowhere-differentiable function; Schrödinger equation; local Lipschitz-Hölder smoothness exponent

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