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Satisfiability with index dependency. (English) Zbl 1310.68114

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Summary: We study the Boolean Satisfiability Problem (SAT) restricted on input formulas for which there are linear arithmetic constraints imposed on the indices of variables occurring in the same clause. This can be seen as a structural counterpart of Schaefer's dichotomy theorem which studies the SAT problem with additional constraints on the assigned values of variables in the same clause. More precisely, let k-SAT$(m, \mathcal{A})$ denote the SAT problem restricted on instances of k-CNF formulas, in every clause of which the indices of the last $k-m$ variables are totally decided by the first m ones through some linear equations chosen from $\mathcal{A}$. For example, if $\mathcal{A}$ contains $i_3 = i_1 + 2i_2$ and $i_4 = i_2 - i_1 + 1$, then a clause of the input to 4-SAT$(2, \mathcal{A})$ has the form $y_{i_1} \vee y_{i_2} \vee y_{i_1+2i_2} \vee y_{i_2-i_1+1}$, with y_i being x_i or $\overline{x_i}$. We obtain the following results:

1. If $m \geq 2$, then for any set $\mathcal{A}$ of linear constraints, the restricted problem k-SAT$(m, \mathcal{A})$ is either in P or NP-complete assuming P $\neq$ NP. Moreover, the corresponding #SAT problem is always #P-complete, and the Max-SAT problem does not allow a polynomial time approximation scheme assuming P $\neq$ NP.

2. $m = 1$, that is, in every clause only one index can be chosen freely. In this case, we develop a general framework together with some techniques for designing polynomial-time algorithms for the restricted SAT problems. Using these, we prove that for any $\mathcal{A}$, #2-SAT$(1, \mathcal{A})$ and Max-2-SAT$(1, \mathcal{A})$ are both polynomial-time solvable, which is in sharp contrast with the hardness results of general #2-SAT and Max-2-SAT. For fixed $k \geq 3$, we obtain a large class of non-trivial constraints $\mathcal{A}$, under which the problems k-SAT$(1, \mathcal{A})$, #k-SAT$(1, \mathcal{A})$ and Max-k-SAT$(1, \mathcal{A})$ can all be solved in polynomial time or quasi-polynomial time.

For the entire collection see [Zbl 1202.68002].

MSC:

68Q25 Analysis of algorithms and problem complexity

Keywords:

Boolean satisfiability problem; index-dependency; index-width; bandwidth; dichotomy

Software:

MAX-2-SAT

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