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**Beyond Sturmian sequences: coding linear trajectories in the regular octagon.** (English)

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The symbolic coding of a linear trajectory in a regular $2n$-gon, where opposite sides are identified, keeps track of the sequence of sides hit by the trajectory. For $n = 2$, the non-periodic cutting sequences are exactly the Sturmian sequences. In the present paper, non-periodic cutting sequences are characterized for the case $n \geq 3$ in terms of a derivation operator and a coherence condition. Here, derivation means that only sandwiched letters are kept, i.e., letters $L$ preceded and followed by the same letter $L'$.

Successive derivations and normalizations of the cutting sequence yield a $2n$-gon Farey expansion (or additive continued fraction expansion) of the angle of the linear trajectory. On the other hand, the continued fraction expansion gives a sequence of substitution operations that generate the cutting seqeunces of trajectories with that slope. In the case of the octagon, a direction has "terminating" Farey expansion if and only if it is in $\mathbb{Q}(\sqrt{2})$. This is similar to the case $n = 2$, where terminating Farey expansions correspond to rational numbers. The factor complexity, i.e., the number of different words of length $k$, of a cutting sequence is bounded by $(n-1)k+1$, and it is equal to $(n-1)k+1$ when the direction is non-terminating.

The algorithm described by the authors can be understood in terms of renormalization of the $2n$-gon translation surface by elements of the Veech group; see also [the authors, Contemp. Math. 532, 29–65 (2010; Zbl 1222.37012)].

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37E35 Flows on surfaces



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