

Godini, G.

An approach to generalizing Banach spaces: Normed almost linear spaces. (English)

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In a previous paper [J. Approximation Theory 43, 338-358 (1985)] the author introduced the notion of a normed almost linear space (shortly nals), which generalizes a real Banach space. A nals X is e.g., the set of all nonempty, bounded and convex subsets A of a real normed linear space E , with the usual addition and multiplication by reals and the norm: $\|A\| = \sup_{a \in A} \|a\|$. It was also defined the dual of a nals, where the functionals are "almost linear", which dual is also a nals.

The aim of this paper is to answer the question whether the dual of a nals $X (\neq \{0\})$ may be reduced to the only zero functional. We remark that the Hahn-Banach theorem is not true, in a nals, if we replace "linear" by "almost linear". In this paper the author did not solve completely this problem, but gives sufficient conditions for an affirmative answer. Another notion studied in this paper is that of a basis of an almost linear space. In contrast with the case of a linear space, an almost linear space, even when it is a nals, does not have a basis in general. There are given some properties of nals with basis.

We are entitled to say that this work is a very interesting one, is full of examples, which give to this work greatest interest for the reader. We remark that, in the mean time, the author has solved in the affirmative the above question on the dual space of a nals as well as all the questions posed in this paper. See Preprint Series in Mathematics, INCREST, No.38 (1985), "On normed almost linear spaces".

Reviewer: [A.Donescu](#)

MSC:

- [46B99](#) Normed linear spaces and Banach spaces; Banach lattices
- [46B15](#) Summability and bases; functional analytic aspects of frames in Banach and Hilbert spaces
- [46B10](#) Duality and reflexivity in normed linear and Banach spaces
- [46A22](#) Theorems of Hahn-Banach type; extension and lifting of functionals and operators

Cited in 1 Document

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normed almost linear space; dual; Hahn-Banach theorem; basis of an almost linear space