**Kartashov, M. V.**

**Boundedness, limits, and stability of solutions of a perturbation of a nonhomogeneous renewal equation on a semiaxis.** (Ukrainian, English) Zbl 1224.60190

Teor. Jmovirn. Mat. Stat. 81, 65-75 (2009); translation in Theory Probab. Math. Stat. 81, 71-83 (2010).

The generalized renewal equation on $\mathbb{R}_+$ generated by a substochastic measure $G$ with an unknown function $x_0$ from the class of Borel functions $B_0 = \{x : \mathbb{R}_+ \to \mathbb{R} \mid \sup_{s\le t} |x(s)| < \infty\ \forall t \ge 0\}$ is the integral equation

$$x_0(t) = y(t) + \int_{[0,t]} x_0(t-s)G(ds), \quad t \ge 0.$$

For a given function $y \in B_0$ this equation has a unique solution $x_0 \in B_0$ represented in the form of a convolution $x_0(t) = y * U(t) = \int_{[0,t]} y(t-s)U(ds)$, where the $\sigma$-finite renewal measure $U$ is defined by $U(B) = \sum_{n\ge 0} G^{*n}(B)$. See, for example [*N. V. Kartashov*, Theory Probab. Math. Stat. 26, 53–67 (1983; Zbl 0515.60086)]. The classical renewal theorem for a probability distribution $G$ of absolutely continuous type states that, for all functions $y \in L_1^0 = \{z \in B_0 \mid \lim_{t\to\infty} z(t) = 0,\ \int_0^\infty |z(s)|ds < \infty\}$, the renewal equation has a unique solution $x_0 \in B_0$, and the following limit exists

$$\lim_{t\to\infty} x_0(t) = m_G^{-1} \int_0^\infty y(s)\,ds, \quad m_G = \int_0^\infty sG(ds).$$

In particular, $\sup_{t\in\mathbb{R}} |x(t)| < \infty$. See, for example [*N. V. Kartashov*, Theory Probab. Math. Stat. 27, 55–64 (1983; Zbl 0521.60088)].

In this article, the author deals with a time-nonhomogeneous perturbation of the classical renewal equation with continuous time on the semiaxis that can be reduced to the Volterra integral equation

$$x(t) = y(t) + \int_0^t x(t-s)F(t,ds), \quad t \in \mathbb{R},$$

for an unknown function $x \in B_0$ and a given function $y \in L_1^0$, where $F(t,ds)$ is a nonnegative bounded (or substochastic) kernel. It is assumed that the latter kernel is approximated in variation by a convolution kernel for large time intervals and that the convolution kernel is generated by a substochastic distribution on the positive semiaxis. Necessary and sufficient conditions are derived for the existence of the limit of a solution of the perturbed equation under the assumption that the corresponding perturbation is small in a certain sense. Estimates for the deviation of the solutions of the perturbed equations from those of the initial equations are found. This investigation is motivated by the problem of the asymptotic behavior of the ruin function for risk processes with a varying intensity of premiums, which were considered, for example, in [*N. V. Kartashov*, Theory Probab. Math. Stat. 60, 53–65 (2000); translation from Teor. Jmovirn. Mat. Stat. 60, 46–58 (1999; Zbl 0955.60072)]. An analogous problem is considered in the author's paper [Theory Probab. Math. Stat. 78, 61–73 (2009); translation from Teor. Jmovirn. Mat. Stat. 78, 54–65 (2008; Zbl 1224.91065)] for the renewal equation on the whole axis. The restriction of the domain to the nonnegative semiaxis allows to obtain more precise results. The proofs in the first part of this paper use some ideas of the paper by *H. Schmidli* [Ann. Appl. Probab. 7, No. 1, 121–133 (1997; Zbl 0876.60072)].

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**MSC:**

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60K05 Renewal theory



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Volterra equation; renewal theorem; transition kernel; regularity; minimal solution; stability

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