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Geometric stability of the cotangent bundle and the universal cover of a projective manifold. (Stabilité géométrique du fibré cotangent et du recouvrement universel d'une variété projective.) (English. French summary) [Zbl 1218.14030](#)
Bull. Soc. Math. Fr. 139, No. 1, 41-74 (2011).

The focus of the article lies on birational positivity properties of the cotangent bundle Ω_X^1 of a connected projective algebraic manifold X . A key role for the considerations is taken by a general theorem about the stability of tensor products of torsion free sheaves on X which reads in the notation of the paper as follows: Let α be a rational ample class in the closed cone of classes of movable curves on X and let \mathcal{E} and \mathcal{F} be α -semi-stable torsion free sheaves on X . Then $\mathcal{E} \otimes \mathcal{F}/\text{torsion}$ is again α -semi-stable.

The algebraic proof of the authors is complemented by an analytic proof, given by M. Toma. The first applications deal with uniruledness criteria related to properties of Ω_X^1 and generalizations of a theorem of Y. Miyaoka which says that X is uniruled if Ω_X^1 is not generically nef. For a proof, see [*N. I. Shepherd-Barron*, *Astérisque*. 211, 103–114 (1992; [Zbl 0809.14034](#))].

The authors prove that the torsion free coherent quotients \mathcal{S} of $(\Omega_X^1)^{\otimes m}$ have pseudo-effective determinants (i.e., $c_1(\det \mathcal{S})$ lies in the closure of the Kähler cone) if X is not uniruled. This result serves in the following as a tool for the proof of several interesting results and as a basis for the statement of conjectures about the geometric stability of Ω_X^1 . Among others, it is shown that the Kodaira dimension $\kappa(X)$ equals $\kappa^+(X) := \max\{\kappa(\det \mathcal{I}) \mid \mathcal{I} \text{ a saturated coherent subsheaf of } \Omega_X^p, 1 \leq p \leq \dim X\}$ if $\kappa(X) \geq \dim X - 3$ or if $\kappa^+(X) = \dim X$. Moreover, $\kappa(X)$ can be estimated from below by any numerically trivial line bundle $L \in \text{Pic}^0(X)$ in the sense that $\kappa(X, mK_X \otimes L) \leq \kappa(X)$, and if $h^0(mK_X \otimes L) \neq 0$ for some $m \in \mathbb{N}$, then $\kappa(X) > 0$ or L is a torsion element.

The proof of this statement is based on the results of *C. Simpson* on jumping loci of numerically trivial line bundles [*Ann. Sci. Éc. Norm. Supér.* (4) 26, No. 3, 361–401 (1993; [Zbl 0798.14005](#))] and on *E. Viehweg's* cyclic covers [*Adv. Stud. Pure Math.* 1, 329–353 (1983; [Zbl 0513.14019](#))]. Other applications are related to the geometry of the universal cover \tilde{X} of X , e.g., if there is no positive dimensional compact subvariety through a general point of \tilde{X} and if $\chi(X, \mathcal{O}_X) \neq 0$, then X is of general type.

Reviewer: [Eberhard Oeljeklaus \(Bremen\)](#)

MSC:

- [14J40](#) n -folds ($n > 4$)
- [32Q26](#) Notions of stability for complex manifolds
- [32J27](#) Compact Kähler manifolds: generalizations, classification
- [14E30](#) Minimal model program (Mori theory, extremal rays)

Cited in **4** Reviews
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Keywords:

uniruledness; Kodaira dimension; pseudo-effective line bundle; rational ample class; semi-stable bundle

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