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Summability of Gabor expansions and Hardy spaces. (English) Zbl 1221.42015

Appl. Comput. Harmon. Anal. 30, No. 3, 288-306 (2011).

The paper deals with Wiener amalgam spaces $W(X,\ell^q)$ on $\mathbb{R}^d$, defined by the condition $\|f\|_{W(X,\ell^q)} = \sum_{k\in\mathbb{Z}^d} \|f_{|[k,k+1)}\|_X^q < \infty$, when $X = L^p, L^{1,\infty}$ and h^p (the local Hardy space), and with the general θ-summability method defined by a function θ in the $W(L^\infty,\ell^1)$-closure of continuous functions which has an integrable Fourier transform $\widehat{\theta}$, with some extra conditions on the derivatives of $\widehat{\theta}$.

If $\sigma_*^\theta h$ is the maximal function of the θ-means for Gabor series, the author obtains boundedness results for $\sigma_*^\theta : h_p \to L^p$ and $\sigma_*^\theta : W(h^p,\ell^\infty) \to W(L^p,\ell^\infty)$, and then the a.e. convergence of the θ-summation method for functions from $W(L^1,\ell^\infty)$.

Reviewer: Joan Cerdà (Barcelona)

MSC:

42B08 Summability in several variables
42B35 Function spaces arising in harmonic analysis

Keywords:

Wiener amalgam spaces; local Hardy spaces; atomic decomposition; θ-summability; Gabor expansions; Gabor frames; time-frequency analysis

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