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PBW filtration and bases for irreducible modules in type A_n . (English) Zbl 1237.17011
Transform. Groups 16, No. 1, 71-89 (2011).

The universal enveloping algebra $U(\mathfrak{a})$ of a finite dimensional Lie algebra \mathfrak{a} has a natural filtration by monomial degrees. Let $\mathfrak{g} = \mathfrak{sl}_n$ with a fixed standard triangular decomposition $\mathfrak{g} = \mathfrak{n}_- \oplus \mathfrak{h} \oplus \mathfrak{n}_+$. For a dominant integral $\lambda \in \mathfrak{h}^*$ let $V(\lambda)$ be the corresponding simple finite dimensional \mathfrak{g} -module with a fixed primitive vector. Applying the components of the degree filtration of $U(\mathfrak{n}_-)$ to the primitive vector induces a filtration on $V(\lambda)$.

The main object of the study in the paper under review is the associated graded vector space $\text{gr } V(\lambda)$. The latter is a cyclic module over the symmetric algebra $S(\mathfrak{n}_-)$ over \mathfrak{n}_- and hence has the form $\mathfrak{n}_-/I(\lambda)$ for some ideal $I(\lambda)$. The first main result of the paper gives a concrete description of $I(\lambda)$. The second main result of the paper gives an explicit monomial basis of $\text{gr } V(\lambda)$, construction of which uses, in particular, combinatorics of Dyck paths.

Reviewer: **Volodymyr Mazorchuk (Uppsala)**

MSC:

- 17B10** Representations of Lie algebras and Lie superalgebras, algebraic theory (weights)
17B67 Kac-Moody (super)algebras; extended affine Lie algebras; toroidal Lie algebras

Cited in **3** Reviews
Cited in **40** Documents

Keywords:

Lie algebra; simple module; basis, PBW filtration; associated graded module; Dyck path

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