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On the symmetry of the ground states of nonlinear Schrödinger equation with potential.
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Summary: We investigate the minimizers of the energy functional

$$\mathcal{E}(u)=\frac{1}{2}\int_{\mathbb{R}^N}|\nabla u|^2\,dx+\frac{1}{2}\int_{\mathbb{R}^N}V|u|^2\,dx-\frac{1}{p+1}\int_{\mathbb{R}^N}b|u|^{p+1}\,dx$$

under the constraint of the L^2-norm. We show that, for the case when the L^2-norm is small, the minimizer is unique and, for the case when the L^2-norm is large, the minimizer concentrates at the maximum point of b and decays exponentially. By this result, we can show that, if V and b are radially symmetric but b does not attain its maximum at the origin, then the symmetry breaking occurs as the L^2-norm increases. Further, we show that, for the case when b has several maximum points, the minimizer concentrates at a point which minimizes a function which is denned by b, V and the unique positive radial solution of $-\Delta\varphi+\varphi-\varphi^p=0$. For the case when V and b are radially symmetric, we show that, if the minimizer concentrates at the origin, then the minimizer is radially symmetric. Further, we construct an energy functional such that the minimizer breaks its symmetry once, but after that it recovers to be symmetric as the L^2-norm increases.

MSC:

35J20 Variational methods for second-order elliptic equations
35J61 Semilinear elliptic equations
35Q40 PDEs in connection with quantum mechanics
81R40 Symmetry breaking in quantum theory

Keywords:

nonlinear Schrödinger equation; ground states; symmetry breaking

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