Newman, Charles M.

The distribution of Lyapunov exponents: Exact results for random matrices. (English)

Zbl 0593.58051
Commun. Math. Phys. 103, 121-126 (1986).

Author's summary: "Simple exact expressions are derived for all the Lyapunov exponents of certain N-dimensional stochastic linear dynamical systems. In the case of the product of independent random matrices, each of which has independent Gaussian entries with mean zero and variance 1/N, the exponents have an exponential distribution as $N \to \infty$. In the case of the time-ordered product integral of $\exp[N^{-1/2}dW]$, where the entries of the $N \times N$ matrix W(t) are independent standard Wiener processes, the exponents are equally spaced for fixed N and thus have a uniform distribution as $N \to \infty$."

Reviewer: G.Warnecke

MSC:

58J70 Invariance and symmetry properties for PDEs on manifolds
60G07 General theory of stochastic processes

Keywords:

Lyapunov exponents; stochastic linear dynamical systems; independent random matrices; Wiener processes

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