

**Kąkol, Jerzy; Kubiś, Wiesław; López-Pellicer, Manuel**

**Descriptive topology in selected topics of functional analysis.** (English) Zbl 1231.46002

*Developments in Mathematics* 24. Berlin: Springer (ISBN 978-1-4614-0528-3/hbk; 978-1-4614-0529-0/ebook). xii, 493 p. (2011).

The book summarizes many results which describe various aspects of the structure of topological vector spaces (some results concern topological groups or topological spaces). Their topological or descriptive topological properties, or relations to such properties of underlying topological spaces, duals etc. form the dominant theme. Some chapters are devoted to the study of spaces with rich families of projections and to compact spaces  $K$  with rich families of retractions together with  $C(K)$  spaces.

Most of the chosen material is relatively recent and appears in book form for the first time. The proofs of the main results and of many needed results about topological spaces are included. Together with a rich list of references the book can help in finding and understanding many finer questions concerning various aspects of the structure of subclasses of the class of topological vector spaces. Compared to that the enclosed index of notions seems to be relatively brief and some often used notions cannot be found there explicitly.

The book is divided into twenty chapters. We try to indicate their content by pointing out some of the studied topics. Our point of view is rather random than in some sense optimal.

Chapter 2 is devoted to Baire-type properties. It contains known results on the existence of non-Baire hyperplanes.

Chapters 3-9 summarize known results on analyticity, or  $K$ -analyticity and related notions, and their relations to other topological properties of topological vector spaces. E.g., some closed graph theorems, theorems on angelicity and metrizability are proved. In particular, many known deep results on  $C_p(X)$  spaces are included.

Chapter 11 gives a number of results about the topological structure like tightness, metrizability, angelicity, analyticity, realcompactness in connection to the weak topologies for the class  $\mathfrak{G}$  of spaces. This class, which contains both the (LF) and the (DF) spaces, was introduced by Cascales and Orihuela. Some particular results with simpler proofs appear in Chapter 10.

In Chapters 12 and 13 results on weakly compactly generated spaces, in particular their  $K$ -analyticity and the separable complementation property, can be found. Results about connections to the Lindelöf property, Corson's property (C) are presented.

Chapters 14 to 16 concern various sequential properties, in particular the Fréchet-Urysohn property of  $C_p(X)$ ,  $C_c(X)$ , or of locally convex spaces in the class  $\mathfrak{G}$ .

In Chapters 17 to 20 the Banach spaces with rich families of projections (projectional resolutions of the identity, projectional skeletons, separable complementation property) are studied. The technique of applying countable submodels is described and used. Chapter 18 is devoted to the particular class of spaces  $C(K)$  for linearly ordered compact  $K$ 's, Chapter 19 is devoted to a deep study of compact spaces, families of retractions and corresponding  $C(K)$  spaces. Chapter 20 presents a construction under the continuum hypothesis of a complementably universal space for the spaces of density  $\aleph_1$  which possess a projectional resolution of the identity.

Reviewer: Petr Holický (Praha)

**MSC:**

- 46-02 Research exposition (monographs, survey articles) pertaining to functional analysis
- 54-02 Research exposition (monographs, survey articles) pertaining to general topology
- 46B26 Nonseparable Banach spaces
- 54C35 Function spaces in general topology
- 54H05 Descriptive set theory (topological aspects of Borel, analytic, projective, etc. sets)

Cited in **4** Reviews  
Cited in **55** Documents

**Keywords:**

topological vector spaces; closed graph theorems; separable complementation property; projectional resolution of the identity; projectional skeletons; Corson's property (C); Baire-type properties; angelicity; analyticity; web-compact spaces; realcompact spaces; metrizable; Fréchet-Urysohn property; tightness; continuous retractive sequences; retractional skeletons

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