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Pseudo-periodic maps and degeneration of Riemann surfaces. (English) Zbl 1239.57001

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The present monograph consists of two parts. In the first part a class of pseudo-periodic maps resp. mapping classes of closed surfaces is studied and classified up to conjugation (those of “negative twist”). This is motivated also by the second part on the topology of degenerating families of Riemann surfaces over a disk, nonsingular outside the origin; in fact, the monodromy around the singular fiber of such a degenerating family is a pseudo-periodic map of negative twist (as proved in a paper by *C. J. Earle* and *P. L. Sipe* [“Families of Riemann surfaces over the punctured disk”, *Pac. J. Math.* 150, No. 1, 79–96 (1991; [Zbl 0734.30039](#)])).

In a paper from 1944, a sequel of his celebrated paper on the “Nielsen realization problem” from 1942 on the realization of periodic mapping classes by periodic maps, Nielsen studied a type of surface mapping classes which he called of “algebraically finite type” [“Surface transformation classes of algebraically finite type”, *Danske Vid. Selsk., Mat.-Fys. Medd.* 21, No. 2, 89 p. (1944; [Zbl 0063.05952](#)); *Collected mathematical papers. Volume 1 (1913–1932). Volume 2 (1932–1955)*. Ed. for the Danish Mathematical Society by Vagn Lundsgaard Hansen. Boston-Basel-Stuttgart: Birkhäuser (1986; [Zbl 0609.01050](#))]. This means that there is a disjoint union of simple closed curves on the surface (the “cut curves”), invariant under a representing homeomorphism f of the mapping class, such that the restriction of f to the complement of the cut curves is isotopic to a periodic map; in more recent terminology, such a mapping class is reducible with finite order components (in the trichotomy pseudo-Anosov, reducible and finite order for surface mapping classes). These are exactly the pseudo-periodic maps resp. mapping classes of the present monograph. Nielsen defined certain invariants of such pseudo-periodic mapping classes, among them a screw number around cut curves, and asserted that these invariants were a complete set of invariants up to conjugacy. However, as the authors note, “his proof of this assertion was rather vague, and we need an invariant (the action of the monodromy on the partition graph) which he did not mention explicitly”.

A pseudo-periodic map is of negative twist if the screw numbers about a certain system of cut curves are all negative. The main result of the first part of the present monograph is then the theorem that pseudo-periodic maps of negative twist are classified, up to conjugation, by the Nielsen invariants and the action on the partition graph. An important ingredient in the proof is the concept of a generalized quotient of a pseudo-periodic map; this plays an important role also in the second part of the monograph since the base space of the generalized quotient of a pseudo-periodic map of negative twist is homeomorphic to the singular fiber of a 1-parameter family of Riemann surfaces with this monodromy. The main result of the second part is the theorem that the topological types of minimal degenerating families of Riemann surfaces of genus $g \geq 2$ over the disk, nonsingular outside the origin, are classified via monodromy by the conjugacy classes of pseudo-periodic maps of negative twist in the mapping class group of genus g .

The main part of the present work was completed in 1991 and announced in a note in [“Pseudo-periodic homeomorphisms and degeneration of Riemann surfaces”, *Bull. Am. Math. Soc., New Ser.* 30, No. 1, 70–75 (1994; [Zbl 0797.30036](#))]. In a recent appendix to the introduction, the authors comment on this delay of publication of two decades, on various connections with related work (such as isolated singular points of complex hypersurfaces, Milnor fiberings, Waldhausen’s graph manifolds), some recent developments (a list of all singular fibers in degenerating families of genus 3, on the basis of the present work), as well as on the difficulties of a general classification of pseudo-periodic maps (i.e., without the negative twist hypothesis). With few exceptions, the exposition is self-contained, elementary and written with great care for details; as the authors note, this makes arguments and proofs quite long, but certainly easy and pleasant to read.

Reviewer: [Bruno Zimmermann \(Trieste\)](#)

MSC:

- 57-02 Research exposition (monographs, survey articles) pertaining to manifolds and cell complexes
- 57M60 Group actions on manifolds and cell complexes in low dimensions
- 30F99 Riemann surfaces

Cited in 1 Review Cited in 5 Documents

Keywords:

reducible mapping class of a surface; pseudo-periodic map; degenerating family of Riemann surfaces over a disk

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