Bourbaki, N.

Within N. Bourbaki’s fundamental and encyclopedic work “Éléments de Mathématique”, Book II is entitled “Algèbre” and comprises ten separate chapters. These chapters were successively published between 1943 and 1980, originally in French. Chapters 1-7 also have been translated into English (by P. M. Cohn and J. Howie). This English translation was published by Springer Verlag in 1980, and that in two volumes. However, the remaining three Chapters 8, 9 and 10, which appeared separately and successively between 1958 and 1980, have never been translated into English, irrespective of their immediate translations into Russian after the publication of the French originals. Having been out of print for several decades, also these three later chapters of N. Bourbaki’s book “Algèbre” have finally been made available again, thanks to a recent reprinting program for all French originals launched by Springer Verlag.

With the exception of Chapter 8, all parts of Book II were published as faithful reproductions of the French originals in 2007.

The volume under review is exactly that remaining Chapter 8 of N. Bourbaki’s “Algèbre”, the first edition of which appeared in 1958 as a small booklet of 189 pages [N. Bourbaki, Éléments de mathématique. XXIII. Première partie: Les structures fondamentales de l’analyse. Livre II: Algèbre. Chap. 8: Modules et anneaux semi-simples, Actualités scientifiques et industrielles. 1261. Paris: Hermann & Cie. (1958; Zbl 0102.27203)]. A very extensive review of this book on semi-simple rings and modules was provided by the famous German algebraist Wolfgang Krull in 1958, and we refer to this historical document of his (Zbl 0102.27203) utmost reverentially.

The current new edition of this volume is by far more than just a reprint of the original from 54 years ago. In contrast to the other chapters, this Chapter 8 has been completely revised, enlarged and updated. In fact, 300 pages have been added, where the number of sections has been increased from 13 (in the original from 1958) to 21 (in the present edition). Also, three additional appendices enrich the present profound revision substantially.

Now as before, this book is intended as a comprehensive exposition of the theory of semi-simple rings and modules, with special emphasis on the Noetherian and Artinian cases. As for the precise contents of the text under review, the material is organized in twenty-one chapters and four appendices, each of which is subdivided into several subsections.


– App. 1. Algebras without unit element; App. 2. Determinants over division algebras; App. 3. Hilbert’s Nullstellensatz; App. 4. Traces of endomorphisms of finite rank.

Each section ends with a large collection of related exercises in the typical Bourbaki-style, thereby providing a wealth of additional concepts, methods, techniques, results, and even theories in the context of the book.

Now as before, the historical note at the end of the book briefly describes the historical developments in this area of algebra until around 1930, without any additions or up-datings.

As one can see from the table of contents of the present new edition of this classic by N. Bourbaki, the entire treatise has been radically revised, re-arranged, modernized, systematized, and appropriately
supplemented. In fact, this version of the book appears as a profound contemporary primer on the subject of semi-simple modules, rings, and algebras, together with their allied topics of Grothendieck groups, Brauer groups, Morita equivalence, and representation theory. Certainly, it has been both a splendid idea and a great undertaking to rewrite N. Bourbaki’s classic Chapter 8 of Book II of the “Elements of Mathematics” in such excellent a manner, very much so to the benefit of further generations of mathematicians.

Reviewer: Werner Kleinert (Berlin)

MSC:

16-02 Research exposition (monographs, survey articles) pertaining to associative rings and algebras
01A75 Collected or selected works; reprints or translations of classics
00A05 Mathematics in general
16P20 Artinian rings and modules (associative rings and algebras)
16P40 Noetherian rings and modules (associative rings and algebras)
16D60 Simple and semisimple modules, primitive rings and ideals in associative algebras
16N20 Jacobson radical, quasimultiplication
16K50 Brauer groups (algebraic aspects)
16E20 Grothendieck groups, K-theory, etc.
16D90 Module categories in associative algebras

Keywords:
simple modules; semisimple modules; simple rings; semisimple rings; simple algebras; semisimple algebras; Jacobson radical; Brauer groups; Grothendieck groups; Morita equivalences; representation theory; chain conditions

Full Text: DOI