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Strong $\Delta^0_2$ categoricity. (English) Zbl 0596.03043

The notion of strong $\Delta^0_2$ stability for recursive structures is first introduced and discussed. A recursive structure $\mathfrak{A}$ is strongly $\Delta^0_2$ stable if there is a total $\Delta^0_2$ function $f$ on $A \times \mathbb{N}$ such that for every recursive structure $\mathfrak{B}$, every possible isomorphism from $\mathfrak{A}$ to $\mathfrak{B}$ is $f(a,n)$ for some $n$. This notion lies between that of recursive stability, previously studied by Goncharov and that of $\Delta^0_2$-stability, studied by Ash. The analogous notion of strong $\Delta^0_2$-categoricity is then also defined.

Several useful-looking examples are produced to show that various combinations of properties can occur, while several questions are posed which remain unanswered. The main question asked is whether there is a natural syntactical characterization (under reasonable assumptions) of the strongly $\Delta^0_2$-categorical recursive structures, along the lines of those obtained by the authors respectively for recursive categoricity and $\Delta^0_2$-categoricity.

The corresponding question is also asked for strong $\Delta^0_2$-stability, although the possibility may be thought to be precluded in this case by the result proved here that a 1-recursive structure is strongly $\Delta^0_2$-stable if and only if it is recursively stable. By contrast, an example shows that a 2-recursive structure may be strongly $\Delta^0_2$-categorical but not recursively categorical.

MSC:
03D45 Theory of numerations, effectively presented structures
03D25 Recursively (computably) enumerable sets and degrees
03C57 Computable structure theory, computable model theory
03D55 Hierarchies of computability and definability

Keywords:
stability; recursive structures; recursive categoricity

Full Text: DOI EuDML

References:

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