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On extension of submeasures. (English) Zbl 0596.28002

Math. Slovaca 34, 265-271 (1984).

Let $\mathcal{R}$ be a ring of subsets of a nonempty set T. According to Definition 1 in an earlier paper by the author [Diss. Math. 112, 35 p. (1974; Zbl 0292.28001)] a set function $\mu:\mathcal{R}\to[0,+\infty)$ is a submeasure if it is monotone, continuous ($A_n\searrow\mathcal{R}0\Rightarrow\mu(A_n)\to0$), and subadditively continuous (for all $A\in\mathcal{R}$ and all $\epsilon>0$ there exists $\delta>0$ such that $B\in\mathcal{R},\mu(B)<\delta\Rightarrow\mu(A)-\epsilon\le\mu(A-B)\le\mu(A)\le\mu(A\cup B)\le\mu(A)+\epsilon$). The property of being subadditively continuous is equivalent to the following property: $A,A_n\in\mathcal{R},n=1,2,\ldots$ and $\mu(A_n\Delta A)\to0\Rightarrow\mu(A_n)\to\mu(A)$. A set function $\mu:\mathcal{R}\to[0,+\infty]$ is exhaustive if $\mu(A_n)\to0$ for each infinite sequence $A_n\in\mathcal{R},n=1,2,\ldots$, of pairwise disjoint sets. In Theorem 18 in the above-cited paper [op. cit.], the author has proved that a uniform, subadditive or additive submeasure $\mu:\mathcal{R}\to[0,+\infty)$ has a unique extension of the same type to σ ($\mathcal{R}$)- the σ-ring generated by $\mathcal{R}$- if and only if it is exhaustive. Two additional, rather clumsy, conditions were needed to obtain the extension theorem for nonuniform submeasures. In this note, using a more transparent approach he shows that these conditions may be replaced by the following: (i) For each $\epsilon>0$ there is a $\delta>0$ such that $A,B\in\mathcal{R}$, $\mu(A),\mu(B)\le\delta$ implies $\mu(A\cup B)<\epsilon$ (pseudometric generating property) and (ii) $A_n\in\mathcal{R},n=1,2,\ldots$, and $\mu(A_n\Delta A_m)\to0$ as $n,m\to\infty$ implies that $\mu(A_n)-\mu(A_m)\to0$ as $n,m\to\infty$.

MSC:

28A12 Contents, measures, outer measures, capacities

28A10 Real- or complex-valued set functions

Keywords:

additive submeasure; unique extension

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