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Gelfand-Tsetlin polytopes and Feigin-Fourier-Littelmann-Vinberg polytopes as marked poset polytopes. (English) Zbl 1234.52009

J. Comb. Theory, Ser. A 118, No. 8, 2454-2462 (2011).

Summary: *R. P. Stanley* [Discrete Comput. Geom. 1, 9–23 (1986; Zbl 0595.52008)] showed how a finite partially ordered set gives rise to two polytopes, called the order polytope and chain polytope, which have the same Ehrhart polynomial despite being quite different combinatorially. We generalize his result to a wider family of polytopes constructed from a poset P with integers assigned to some of its elements.

Through this construction, we explain combinatorially the relationship between the Gelfand-Tsetlin polytopes [*I. Gelfand* and *M. Tsetlin*, Dokl. Akad. Nauk. SSSR 71, 825–828 (1950; Zbl 0037.15301)] and the Feigin-Fourier-Littelmann-Vinberg polytopes [*E. Feigin, Gh. Fourier, P. Littelmann*, Int. Math. Res. Not. 2011, No. 24, 5760–5784 (2011; Zbl 1233.17007); *E. Vinberg*, On some canonical bases of representation spaces of simple Lie algebras, Conference talk, Bielefeld (2005)], which arise in the representation theory of the special linear Lie algebra. We then use the generalized Gelfand-Tsetlin polytopes of *A. D. Berenshtejn* and *A. V. Zelevinskij* [J. Geom. Phys. 5, No. 3, 453–472 (1988; Zbl 0712.17006)] to propose conjectural analogues of the Feigin-Fourier-Littelmann-Vinberg polytopes corresponding to the symplectic and odd orthogonal Lie algebras.

MSC:

52B20 Lattice polytopes in convex geometry (including relations with commutative algebra and algebraic geometry)
52B12 Special polytopes (linear programming, centrally symmetric, etc.)
05E10 Combinatorial aspects of representation theory

Keywords:

order polytope; chain polytope; Gelfand-Tsetlin polytope; Lie algebra; irreducible representation; Ehrhart polynomial

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