

Losert, Viktor

The Borel property for simple Riesz means. (English) Zbl 0597.10051

Monatsh. Math. 102, 217-226 (1986).

Let $P = (p_n)$ be a sequence of positive real numbers. The complex sequence (α_n) is called P-summable, if $(1/P_N) \sum_{n=1}^N p_n \alpha_n$ converges, where $P_N = \sum_{n=1}^N p_n$. Assuming that P is increasing, it is shown that almost all sequences (with respect to any probability measure on \mathbb{C}) are P-summable iff "Hill's condition" (H) holds, i.e. $\sum_{n=1}^{\infty} \exp(-\delta/a_n) < \infty$ for all $\delta > 0$, where $a_n = P_n^{-2} \sum_{k=1}^n p_k^2$. The sufficiency of (H) is due to *J. D. Hill* [Pac. J. Math. 1, 399-409 (1951; Zbl 0043.286)]. For arbitrary (non-monotonic) weights, (H) is still sufficient, but not necessary. We give a related necessary and sufficient condition.

MSC:

11K06 General theory of distribution modulo 1

40D09 Structure of summability fields

Keywords:

simple Riesz means; Borel property; uniform distribution

Full Text: [DOI](#) [EuDML](#)

References:

- Hi1 Hill, J. D.: The Borel property of summability methods. Pacific J.1, 399-409 (1951). · [Zbl 0043.28603](#)
- Hi2 Hill, J. D.: Remarks on the Borel property. Pacific J.4, 227-242 (1954). · [Zbl 0057.29301](#)
- Hl1 Hlawka, E.: Folgen auf kompakten Räumen. Abh. Math. Sem. Hamburg20, 223-241 (1956). · [Zbl 0072.05701](#)
- Hl2 Hlawka, E.: Theorie der Gleichverteilung. Mannheim: B. I. 1979. · [Zbl 0406.10001](#)
- KN Kuipers, L., Niederreiter, H.: Uniform Distribution of Sequences, New York: Wiley. 1974. · [Zbl 0281.10001](#)
- L Loève, M.: Probability Theory I. 4th Ed. New York-Heidelberg-Berlin: Springer. 1977.
- M Martikainen, A. I.: On necessary and sufficient conditions for the strong law of large numbers. Theory Probab. Appl.24, 813-820 (1980). · [Zbl 0441.60026](#) · [doi:10.1137/1124093](#)
- SZ Salem, R., Zygmund, A.: Some properties of trigonometric series whose terms have random signs. Acta Math.91, 245-301 (1954). · [Zbl 0056.29001](#) · [doi:10.1007/BF02393433](#)
- T Tsuji, M.: On the uniform distribution of numbers mod 1. J. Math. Soc. Japan4, 313-322 (1952). · [Zbl 0048.03302](#) · [doi:10.2969/jmsj/00430313](#)
- ZB Zeller, K., Beekmann, W.: Theorie der Limitierungsverfahren. Berlin-Heidelberg-New York: Springer. 1970. · [Zbl 0199.11301](#)

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