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On partially ordered groups of locally finite length. (English) Zbl 0598.06009
Math. Slovaca 36, 119-127 (1986).

For any cardinal $\alpha \geq 2$ denote by \mathfrak{G}_α the class of all abelian po-groups G such that the poset (G, \leq) is directed, of locally finite length, all saturated chains from a to b ($a < b$) have the same length and the subset of all elements in G covering zero has cardinality α . E.g., the direct sum D_α of α copies of $\mathbb{Z}(+)$ with the natural linear order belongs to \mathfrak{G}_α . Let $f = \alpha_1 x_1 + \dots + \alpha_n x_n$, $\alpha_i \in \mathbb{Z}$, be any nonzero element in the free abelian group F_α with the set $X = \{x_i \mid i \in I\}$ of free generators. Putting $f > 0$ for elements $f \in F_\alpha$ with all $\alpha_i > 0$ we get a po-group F'_α isomorphic to $D_\alpha \in \mathfrak{G}_\alpha$. The author proves that for a po-group G to be in \mathfrak{G}_α is equivalent to be isomorphic with F'_α/K where $K \leq F'_\alpha$ satisfies the conditions: (i) if x_1 and x_2 are distinct elements of X then $x_1 - x_2 \notin K$, and (ii) $K \leq \text{Ker}\psi$ where the mapping $\psi : F'_\alpha \rightarrow \mathbb{Z}$ is defined by the rule $\psi(f) = \sum_i \alpha_i$. It is proved also that for each cardinal $\alpha \geq 2$ there exists an infinite set of non-isomorphic po-groups belonging to \mathfrak{G}_α .

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MSC:

06F20 Ordered abelian groups, Riesz groups, ordered linear spaces

Keywords:

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References:

- [1] BENADO M.: Sur la théorie de la divisibilité. Acad. R. P. Romine. Bul. Sti. Sect. Sti. Mat. Fiz. 6, 1954, 263-270. · [Zbl 0057.25301](#)
- [2] BENADO M.: Les ensembles partiellement ordonnés et la théorème de raffinement de Schreier, II. Czech. Mat. J. 5 (80), 1955, 308-344. · [Zbl 0068.25902](#)
- [3] ФУКС Л.: Частично упорядоченные алгебраические системы. Москва 1965.
- [4] McALISTER D. B.: On multilattice groups. Proc. Camb. Phil. Soc. 1965, 61, 621-638. · [Zbl 0135.06203](#)
- [5] McALISTER D. B.: On multilattice groups II. Proc. Camb. Phil. Soc. 1966, 62, 149-164. · [Zbl 0138.02702](#)
- [6] ШИМБИРЄВА Є. П.: К теории частично упорядоченных групп. Мат. Сборник, 20, 1947, 145-178.

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