Heins, Maurice

Selected topics in the classical theory of functions of a complex variable. (English)

Zbl 1226.30001
Athena Series: Selected Topics in Mathematics. New York, NY: Holt, Rinehart and Winston. xi, 160 p. (1962).

This is an excellent book on the geometric theory of functions. It is intended for mathematics students who have had completed or are completing a first course on the theory of a complex variable. The object is to present to such students a number of important topics, such as the big Picard theorem, the Riemann mapping theorem and the Fatou radial limit theorem from the classical theory of functions. However, the author's methods are fresh and systematic, and expositions are self-contained and rigorous. In this text, a knowledge of the Lebesgue theory of integration is not presupposed, although the author points out that this theory plays an indispensable role in the study of many important classes of functions. What is needed from real-variable theory is developed in this text. The author adopts a fresh interesting method of exposition which presents certain topics via (finite) sequences of exercises rendered accessible by adequate indications of essential difficulties. The author states modestly that the book [Aufgaben und Lehrsätze aus der Analysis. Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen. Bd. XIX, XX. Berlin: Julius Springer (1925; JFM 51.0173.01)] of *G. Pólya* and *G. Szegö* is a model for this method. The reviewer notes that the author has made many important contributions to complex analysis by applying the theory of subharmonic functions systematically, and a large number of theorems and methods in this text were obtained by the author himself.

This book consists of six chapters and an appendix. In Chapter 1, the reviewer is very much interested in nice explanations of the notion of simple connectivity, analytic logarithm and the relation between Fourier series and analytic functions, together with important problems. In Chapter 2, covering properties of meromorphic functions, argument principle, local analysis of an analytic function, lower semicontinuity of the valence function, boundary values of a meromorphic function, boundary-preserving maps and the Riemann mapping theorem are discussed; here the use of sequences of exercises is made in the proof of the Riemann mapping theorem based on Koebe's method. In Chapter 3, the Picard theorem, the Bloch theorem, the theorem of Schottky and the proof of the big Picard theorem are stated. In Chapter 4, harmonic and subharmonic functions, and Chapter 5, applications, properties of harmonic functions, the Poisson-Stieltjes integral, the Fatou theorem, properties of subharmonic functions, Perron families, $\beta \geq \frac{1}{4}\sqrt{3}$, where β denotes the Bloch constant, the Dirichlet problem, Green's functions, the Lindelöf principle, the inequality of Milloux-Schmidt, the Phragmén-Lindelöf theorem, Wiman's theorem and a method of Carleman are discussed. As to the Bloch constant β, the reviewer refers to the author's recent paper [Nagoya Math. J. 21, 1–60 (1962; Zbl 0113.05603)] in which he proves that β is actually greater than $\frac{1}{4}\sqrt{3}$. In Chapter 6 on the boundary behavior of the Riemann mapping function for simply-connected Jordan regions, we have a nice exposition of Beckenbach's stronger form of the Cauchy integral theorem. The appendix is devoted to the Riesz representation theorem for positive additive functionals, Lebesgue's theorem on the derivative of a monotone function and the Jordan curve theorem. The reviewer notes that throughout this text the author tries to point out the important fact that complex function theory is organically interrelated with many branches of mathematics.

Reviewer: K. Noshiro (MR0162913)

MSC:

30-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to functions of a complex variable
30Cxx	Geometric function theory