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Consistent SUPG-method for transient transport problems: stability and convergence. (English) Zbl 1227.76047

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Summary: We consider the time/space discretization of the transient advection equation. Discretization in space is performed by the streamline upwind Petrov-Galerkin method and in time we use an $\mathcal{A}$-stable finite difference operator. The formulation is strongly consistent in the sense that the time derivative is included in the stabilization term. Uniform stability of the general formulation is proved under a regularity condition on data, or a moderate inverse CFL-condition that allows for optimal choices of the discretization parameters. Both the backward Euler method (BDF1), the Crank-Nicolson scheme and the second-order backward differentiation formula (BDF2) enter the framework and quasi-optimal convergence is proved for these schemes.

MSC:

76M20	Finite difference methods applied to problems in fluid mechanics
76M10	Finite element methods applied to problems in fluid mechanics
76R99	Diffusion and convection
65M12	Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs

Keywords:

SUPG; time discretization; Crank-Nicolson; backward Euler; stability; convergence

Software:

FreeFem++

Full Text: DOI

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