Zhikov, V. V.

Averaging of functionals of the calculus of variations and elasticity theory. (English. Russian original) Zbl 0599.49031

Math. USSR, Izv. 29, No. 1, 33-66 (1987); translation from Izv. Akad. Nauk SSSR, Ser. Mat. 50, No. 4, 675-710, 877 (1986).

The author begins his discussion with a comment that variational problems of mechanics appear to have increasingly more complex Lagrangians. Some change their properties in different regions, some are singular, some vary pointwise. In general, Lagrangians $f(x,\xi)$ are measurable functions of x and convex in ξ for a.e. $x \in \mathbf{R}^n$. ξ could be a vector or a symmetric matrix $\{\xi_{ij}\}$, which is the case in classical elasticity. With each Lagrangian $f(x,\xi)$ is associated a dual Lagrangian $f^*(x,\xi) = \sup_\eta\{\eta \cdot \xi - f(x,\eta)\}$. This Young-type duality is used in a systematic manner to derive averaging results for nonlinear variational problems, particularly for the so-called regular Lagrangians obeying the inequalities $c_1|\xi|^\alpha - c_0 \le f(x,\xi) \le c_2|\xi|^\alpha + c_0$, $c_i > 0$, $\alpha > 1$.

These averaging methods are applied to random Lagrangians, to problems in plasticity with singular Lagrangians, and to Lagrangians representing the behavior of composite materials, or materials with inclusions. This is a development that follows the ingenious applications of the Young-Fenchel duality developed by Soviet authors such as V. L. Berdichevskiĭ, who has written a book "Variational principles of continuum mechanics" (Russian), "Nauka", Moscow (1983)] and a series of articles.

This approach competes with regularization ideas in deciding when averaging is legal, particularly in problems dealing with critical loads and the corresponding designs of elastic systems.

Reviewer: Vadim Komkov (MR 88a:49026)

MSC:

49S05 Variational principles of physics
74S30 Other numerical methods in solid mechanics (MSC2010)

Full Text: DOI