Morgan, John W. (ed.); **Bass, Hyman (ed.)**
The Smith conjecture. (English) Zbl 0599.57001
Pure and Applied Mathematics, 112. Orlando etc.: Academic Press, Inc. XV, 243 p. $ 49.50; £35.00 (1984).

It may be conjectured that any finite group of orientation-preserving diffeomorphisms acting on the 3-sphere, the 3-dimensional Euclidean space, or more generally any geometric 3-manifold, is conjugate to a linear group, resp. a group of isometries. All attempts undertaken so far to attack this problem distinguish between free and non-free actions, and both of these cases are long known to be deep and difficult. The special case of a cyclic group acting (non-freely) on the 3-sphere is classical and known as the Smith conjecture.

The first chapters of the book under review describe the history of the Smith conjecture and further include a brief sketch of its proof. It is here that the reader learns to his surprise that the proof of the longstanding Smith conjecture is rather simple - provided, however, he agrees to accept three deep techniques in 3-manifold theory: the hyperbolization of 3-manifolds, the arboreal technique and the minimal surface technique. As a matter of fact the Smith conjecture simply appears as a by-product of these techniques and, despite its title, the purpose of the book under review is to report on the invention of these techniques and to popularize some of the ideas behind them.

The book itself is an outgrowth of a conference held in 1979 at Columbia University to celebrate the solution of the Smith conjecture, and it reflects the state of affairs in those days. In the meantime not only the above mentioned techniques proved to be very powerful in 3-manifold theory indeed, but also some alternative approaches to the results dealt with in this book have been discovered. In particular, Thurston himself announced a generalization of his results to a geometrization theorem for orbifolds. This not only leads to a conceptually different proof of the Smith conjecture, but more generally to a solution of the conjecture indicated in the beginning for all those finite group actions on geometric 3-manifolds which have one-dimensional fixed point sets.

Technically, the book under review is a collection of reports and research articles written by various authors and I now report on these articles separately.

Reviewer: K.Johannson

MSC:

57-06	Proceedings, conferences, collections, etc. pertaining to manifolds and cell complexes
00Bxx	Conference proceedings and collections of articles

Cited in **7** Reviews
Cited in **75** Documents

Keywords:

cyclic group acting on the 3-sphere; group of isometries; non-free actions; Smith conjecture; hyperbolization of 3-manifolds; finite group actions on geometric 3-manifolds; one-dimensional fixed point sets