Feigin, Evgeny; **Fourier, Ghislain**; **Littelmann, Peter**

PBW filtration and bases for symplectic Lie algebras. (English) Zbl 1233.17007

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Consider the Lie algebra $\mathfrak{sp}_{2n}$ with a fixed triangular decomposition $\mathfrak{n}_- \oplus \mathfrak{h} \oplus \mathfrak{n}_+$. For a dominant $\lambda \in \mathfrak{h}^*$ let $V(\lambda)$ be the simple $\mathfrak{sp}_{2n}$-module with highest weight λ. As an $\mathfrak{n}_-$-module, the module $V(\lambda)$ is a quotient of $U(\mathfrak{n}_-)$. The degree filtration on $U(\mathfrak{n}_-)$ gives rise to a filtration on $V(\lambda)$ and the main object of the study in the paper under review is the associated graded space $\mathrm{gr}V(\lambda)$, which is a quotient of $S(\mathfrak{n}_-)$ modulo some ideal $I(\lambda)$. The first main result of the paper gives an explicit finite dimensional subspace generating $I(\lambda)$ as an $S(\mathfrak{n}_-)$-module. The second main result provides an explicit basis for $\mathrm{gr}V(\lambda)$. As a corollary the authors derive a graded combinatorial formula for the character of $V(\lambda)$ and obtain a new class of bases for the latter module.

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MSC:

17B10 Representations of Lie algebras and Lie superalgebras, algebraic theory (weights)
17B20 Simple, semisimple, reductive (super)algebras

Keywords:

PBW filtration; symplectic Lie algebra; basis; highest weight module; character

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