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Motives of some acyclic varieties. (English) Zbl 1244.14015

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A smooth connected complex variety X is said to be $\mathbb{Z}$-acyclic (resp. $\mathbb{Q}$-acyclic) if $X(\mathbb{C})$, viewed as a a complex manifold, has trivial reduced integral (resp. rational) singular homology. Let $DM_{gm}(\mathbb{C})_{\mathbb{Q}}$ be Voevodsky's triangulated category of motives with $\mathbb{Q}$-coefficients. Then, assuming some "standard conjectures"' about motives, the Hodge conjecture predicts that the Hodge realization functor from $DM_{gm}(\mathbb{C})_{\mathbb{Q}}$ to a derived category of Hodge structures is conservative.

Therefore a $\mathbb{Q}$-acyclic smooth complex variety should conjecturally have a trivial rational motive.

In the case $\dim X = 2$, due to results by Fujita, Gurjar, Pradeep and Shastry, a $\mathbb{Z}$-acyclic (resp. $\mathbb{Q}$-acyclic) smooth complex surface X is rational and affine and there exists an open immersion $X \to \widetilde{X}$, with $\widetilde{X}$ a smooth projective surface such that the boundary $\widetilde{X} - X$ is a simple normal crossing divisor and each irreducible component of it is a rational curve. Using this result, the author in this paper proves the following theorem, which gives some evidence to the conjecture above, in the case of surfaces.

Theorem 1. If X is a $\mathbb{Z}$-acyclic (resp. $\mathbb{Q}$-acyclic) smooth complex variety of dimension 2, then the canonical morphism $M(X) \to \mathbb{Z}$ (resp. $M(X) \to \mathbb{Q}$) is an isomorphism in $DM_{gm}(\mathbb{C})$ (resp.$DM_{gm}(\mathbb{C})_{\mathbb{Q}}$).

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MSC:

14F42	Motivic cohomology; motivic homotopy theory
14R05	Classification of affine varieties
19E15	Algebraic cycles and motivic cohomology (K-theoretic aspects)

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