Benson, James D.; **Chicone, Carmen C.**; **Critser, John K.**

A general model for the dynamics of cell volume, global stability, and optimal control. (English) Zbl 1230.92014

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Summary: Cell volume and concentration regulation in the presence of changing extracellular environments has been studied for centuries, and recently a general non-dimensional model was introduced that encompassed solute and solvent transmembrane flux for a wide variety of solutes and flux mechanisms. Moreover, in many biological applications it is of considerable interest to understand optimal controls for both volume and solute concentrations. We examine a natural extension of this general model to an arbitrary number of solutes or solute pathways, show that this system is globally asymptotically stable and controllable, define necessary conditions for time-optimal controls in the arbitrary-solute case, and using a theorem of *V.G. Boltyanskij* [SIAM J. Control 4, 326–362 (1966; Zbl 0143.32004)] prove sufficient conditions for these controls in the commonly encountered two-solute case.

MSC:

92C37	Cell biology
49N90	Applications of optimal control and differential games
49K15	Optimality conditions for problems involving ordinary differential equations
34D23	Global stability of solutions to ordinary differential equations

Keywords:

Cellular mass transport; Optimization; Stability; Cryobiology; Sufficiency theorem

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