Ekeland, I.; **Hofer, H.**

Subharmonics for convex nonautonomous Hamiltonian systems. (English) Zbl 0601.58035

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Denote by J the standard symplectic matrix in $\mathbb{R}^{2n}$ and let $H \in C^2(\mathbb{R} \times \mathbb{R}^{2n}, \mathbb{R})$ be T-periodic in the first variable. In this paper we investigate the existence of kT-periodic solutions of the time dependent Hamiltonian system

$$(HS)_k \quad -J\dot{x} = H'(t,x), \quad x(0) = x(kT),$$

where $k \in \mathbb{N}$. A solution of $(HS)_k$ for $k \geq 2$ is called a subharmonic. Clearly, a solution of $(HS)_k$ will also be a solution of $(HS)_{2k}$, $(HS)_{3k}$, etc. We show under a convexity assumption on H and a suitable asymptotic behaviour of H that for every $k \in \mathbb{N}$ there is a solution x_k of $(HS)_k$ such that the x_k, $k \in \mathbb{N}$, are pairwise geometrically distinct.

MSC:

37J45 Periodic, homoclinic and heteroclinic orbits; variational methods, degree-theoretic methods (MSC2010)
34C25 Periodic solutions to ordinary differential equations
58E05 Abstract critical point theory (Morse theory, Lyusternik-Shnirel'man theory, etc.) in infinite-dimensional spaces

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References:

[1] Clarke, Comm. PAM 33 pp 103– (1980)
[2] Clarke, Archive Rat. Mech. An. 78 pp 315– (1982)
[3] Ekeland, J. Diff. Eq. 34 pp 523– (1973)
[4] An index theory for periodic solutions of convex Hamiltonian systems. Proceedings AMS Summer Institute on Nonlinear Functional Analysis at Berkeley, 1983, (to appear).
[5] Une théorie de Morse pour les systèmes hamiltoniens convexes. Annales IHP "Analyse non linéaire," 1, 1984, pp. 19–78.
[6] Ekeland, Inventiones Math. 81 pp 155– (1985)
[7] Hofer, Proceedings of Symposia in Pure Mathematics 45 pp 501– (1986)· doi:10.1090/pspum/045.1/843584
[8] Hofer, J. London Math Society 31 pp 566– (1985)
[9] Topological Methods in the Study of Nonlinear Integral Equations. Pergamon Press, 1963.
[10] Mather, Adv. in Math 4 pp 301– (1970)
[11] and The structure of the critical set in the mountain pass theorems. Preprint 1985.
[12] Rabinowitz, Comm. PAM 33 pp 609– (1980)