Carlson, James A.

Polyhedral resolutions of algebraic varieties. (English) Zbl 0602.14012

Trans. Am. Math. Soc. 292, 595-612 (1985).

In this paper, a method of constructing smooth simplicial resolutions which are small is described. (These resolutions are used to compute the mixed Hodge structure of complex projective varieties.)

Let X be an algebraic variety, $\Sigma \subset X$ the singular locus and $\pi : \tilde{X} \to X$ a resolution of singularities. Then the abstract mapping cylinder $C(\pi) = [\tilde{X} \leftarrow \tilde{\Sigma} \to^{\pi} \Sigma]$, $\tilde{\Sigma} = \pi^{-1}(\Sigma)$, is a simplicial space over the unit interval $I^1 = [0, 1]$. Although the singular locus of $X|I^1 := C(\pi)$ may not be empty, it must be of smaller dimension than the singular locus of $X|I^0 := X$. Therefore the main idea is to go on by induction in order to construct a smooth simplicial resolution. Suppose $X|I^p$, $p \geq 1$, has been constructed with $X|\{0\} \times I^{p-1}$ smooth and $AX|I^p$ epimorphic, $(AX)_\sigma = X_\sigma$, $\sigma \notin \{0\} \times I^{p-1}$, and $(AX)_{\{0\}\times\sigma} = im(X_{I^1\times\sigma} \to X_{\{0\}\times\sigma})$. Then one can show that a birational morphism $\pi : (\tilde{\ })|I^p \to AX|I^p$, $(\tilde{\ })$ smooth and epimorphic, exists. Thus it is possible to construct an adequate resolution $\pi_p : \tilde{X}|I^p \to X|I^p$ such that $X|I^{p+1} := C(\pi_p)$ satisfies the two conditions above. At least, this process stops after $n = \dim(X)$ steps and one obtains a smooth simplicial resolution.

Reviewer: M.Heep

MSC:

14E15 Global theory and resolution of singularities (algebro-geometric aspects)
14A10 Varieties and morphisms
55U10 Simplicial sets and complexes in algebraic topology
14C30 Transcendental methods, Hodge theory (algebro-geometric aspects)

Cited in **10** Documents

Keywords:

constructing smooth simplicial resolutions; mixed Hodge structure; resolution of singularities

Full Text: DOI

References:

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[2] Pierre Deligne, Théorie de Hodge. II, Inst. Hautes Études Sci. Publ. Math. 40 (1971), 5 – 57 (French). Pierre Deligne, Théorie de Hodge. III, Inst. Hautes Études Sci. Publ. Math. 44 (1974), 5 – 77 (French).