

**Asok, Aravind**

**Remarks on  $\mathbb{A}^1$ -homotopy groups of smooth toric models.** (English) Zbl 1256.14019  
Math. Res. Lett. 18, No. 2, 353-356 (2011).

In this short note, the author continues his important longstanding project of the classification of homotopy types in the  $\mathbb{A}^1$ -homotopy theory of smooth varieties over a field. Let  $k$  be a field of characteristic 0,  $T$  be a torus over  $k$ , and let  $X$  be a smooth proper variety that is an equivariant compactification of  $T$  over  $k$ . To this data, there is an associated Neron-Severi torus  $T_{NS(X)}$  and a  $T_{NS(X)}$ -torsor  $f : U \rightarrow X$ . The main result of the paper is that this  $T_{NS(X)}$ -torsor  $f : U \rightarrow X$  is an  $\mathbb{A}^1$ -cover. This implies in particular, that the  $\mathbb{A}^1$ -fundamental groups of  $U$  and  $X$  sit in a short exact sequence

$$1 \rightarrow \pi_1^{\mathbb{A}^1}(U, \tilde{x}) \rightarrow \pi_1^{\mathbb{A}^1}(X, x) \rightarrow T_{NS(X)} \rightarrow 1$$

where  $x$  is a given  $k$ -rational point on  $X$  and  $\tilde{x}$  is any lift of  $x$  to  $U$ . Moreover, for  $i > 1$ , there are isomorphisms  $\pi_i^{\mathbb{A}^1}(U, \tilde{x}) \cong \pi_i^{\mathbb{A}^1}(X, x)$ . This extends previous results on torsors isomorphic to  $\mathbb{G}_m^{\times n}$  to the case of general  $k$ -torsors.

Reviewer: Gereon Quick (Münster)

**MSC:**

**14F99** (Co)homology theory in algebraic geometry  
**14M25** Toric varieties, Newton polyhedra, Okounkov bodies

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$\mathbb{A}^1$ -homotopy types; toric varieties

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