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**On differential properties of functions of bounded variation.** (English) Zbl 1299.26036  
*Anal. Math.* 38, No. 1, 1-17 (2012).

Summary: It is established that Karagulyan's exact estimate of the divergence rate of strong integral means of summable functions is extendable to strong means of additive functions of intervals having bounded variation. Furthermore, it is proved that each function defined on  $[0, 1]^n$  with bounded variation in the sense of Hardy has a strong gradient at almost every point (this strengthens the corresponding result of Burkill and Haslam-Jones on the differentiability almost everywhere), whereas the same is not true for functions with bounded variation in the sense of Arzela.

**MSC:**

**26B30** Absolutely continuous real functions of several variables, functions of bounded variation Cited in 1 Document

**Keywords:**

divergence rate; strong means of additive functions with bounded variation; strong gradient; functions with bounded variation in the sense of Hardy and Arzela

**Full Text:** [DOI](#)

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