Bruno, Oscar P.

On a property of ideals of differentiable functions. (English) Zbl 0603.26010

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et $J \subseteq C^\infty(R^n)$ be any ideal. Since a function of the variables $\bar{t} = (t_1, ..., t_n)$ is a function of the variables $(\bar{t}, \overline{x}) = (t_1, ..., t_n, x_1, ..., x_p)$ which does not depend on $\overline{x}$, we have $J \subseteq C^\infty(R^{n+p})$. Of course, J is not an ideal of $C^\infty(R^{n+p})$, but it generates an ideal that we call $J(\bar{t}, \overline{x})$. Consider the following statement (1) on J: "Given any $f \in C^\infty(R^{n+p})$, $f \in J(\bar{t}, \overline{x})$ if and only if for every fixed $a \in R^p$, $f(\bar{t}, \overline{a}) \in J$."

In this paper we show that statement (1) holds for a large class of finitely generated ideals although not for all of them. We say that ideals satisfying statement (1) have line determined extensions. We characterize these ideals to be closed ideals J($\bar{t}$) (in the sense of Whitney) such that for all $p \in N$, the ideal $J(\bar{t}, \overline{x})$ is also closed. Finally, some non-trivial examples are developed.

MSC:

26E10 C^∞-functions, quasi-analytic functions
58C20 Differentiation theory (Gateaux, Fréchet, etc.) on manifolds

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References:

[1] Weil, Géométrie différentielle pp 111– (1953)
[2] DOI: 10.2307/2374046 · Zbl 0483.58003 · doi:10.2307/2374046
[3] Reyes, The L.E.J. Brouwer Centenary Symposium pp 377– (1982)
[4] DOI: 10.2307/2372203 · Zbl 0037.35502 · doi:10.2307/2372203