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On zeroes of some transcendental equations. (English) Zbl 0603.34069

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The equations treated in this paper are of the form (*) $P(z)+Q(z)e^{-Tz}=0$ and arise from the consideration of delay-differential equations with a single delay T, $T\geq 0$. The equation is called stable if all zeroes lie in $Re(z)<0$ and unstable if at least one zero lies in $Re(z)>0$. A stability switch is said to occur if the equation changes from stable to unstable, or vice-versa, as T varies through particular values. The main result is the following theorem.

Assume that: (i) P(z), Q(z) are analytic functions in $Re(z)>\delta$ $(\delta>0)$ which have no common imaginary zero; (ii) the conjugates of P(-iy) and Q(-iy) are P(iy) and Q(iy) for real y; (iii) $P(0)+Q(0)\neq 0$; (iv) there are at most a finite number of zeroes of P(z) $+Q(z)$ in the right half-plane; (v) $F(y)\equiv |P(iy)|^2-|Q(iy)|^2$ for real y, has at most a finite number of real zeroes.

Under these conditions, the following statements are true: (a) If the equation $F(y)=0$ has no positive roots, then if (*) is stable [unstable] at $T=0$ it remains stable [unstable] for all $T\geq 0$.

(b) Suppose that $F(y)=0$ has at least one positive root and each positive root is simple. As T increases, stability switches may occur. There exists a positive T^* such that (*) is unstable for all $T>T^*$. As T varies from 0 to T^*, at most a finite number of stability switches may occur.

MSC:

34K20	Stability theory of functional-differential equations
30C15	Zeros of polynomials, rational functions, and other analytic functions of one complex variable (e.g., zeros of functions with bounded Dirichlet integral)

Keywords:

linear scalar differential-difference equation; transcendental equations; delay-differential equations; stability switches