Brown, M.

Homeomorphisms of two-dimensional manifolds. (English) Zbl 0605.57005

Houston J. Math. 11, 455-469 (1985).

The author introduces and develops some theory for the class of free homeomorphisms: A homeomorphism f of a connected manifold M^m is called free provided that whenever D is an m-disk in M and $f(D) \cap D = \emptyset$ then $f^n(D) \cap D = \emptyset$ for all positive integers n. In the 2-dimensional case a lemma of L. E. J. Brouwer is used to show (Corollary 5.8) that this class of homeomorphisms includes the fixed point free orientation-preserving homeomorphisms of $\mathbb{R}^2$.

More generally, on an orientable 2-manifold it is shown that a homeomorphism is free if and only if it is orientation-preserving and satisfies Brouwer's translation arc property (Corollary 4.5 and Theorem 4.9) - this means that whenever α is an arc from p to $q = f(p)$ in M and $f(\alpha - \{q\}) \cap (\alpha - \{q\}) = \emptyset$ then $f^n(\alpha - \{q\}) \cap (\alpha - \{q\}) = \emptyset$ for all $n > 1$. Some further properties and examples of free homeomorphisms on 2-manifolds are also given. Finally, it is shown that the only free homeomorphism on a manifold of dimension bigger than 2 is the identity homeomorphism (Theorem 6.1).

Reviewer: A.Miller

MSC:

57N05 Topology of the Euclidean 2-space, 2-manifolds (MSC2010)
57S30 Discontinuous groups of transformations
57N15 Topology of the Euclidean n-space, n-manifolds ($4 \leq n \leq \infty$) (MSC2010)
57R50 Differential topological aspects of diffeomorphisms

Keywords:

free homeomorphisms; connected manifold; translation arc property