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0-1 laws of a probability measure on a locally convex space. (English) Zbl 0605.60040

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The authors introduce the following notions of 0-1 laws for a measure μ defined on the σ-algebra generated by the topological dual E' of a locally convex Hausdorff space E resp. on the σ-algebra generated by the open subsets of E.

(0) For every x'$\in E'$, $\mu(x; x'(x) = 0) = 0$ or 1.

(1) p5 For every μ-measurable linear subspace $F \subset E$, $\mu(F) = 0$ or 1.

(2) For every sequence $x'_n \in E'$ and every convex Lusin subspace F of R^∞, $\mu(x; (x'_n(x)) \in F) = 0$ or 1.

(3) For every sequence $x'_n \in E'$, $\mu(x; (x'_n(x)) \in l_\infty) = 0$ or 1.

(4) For every sequence $x'_n \in E'$, $\mu(x; (x'_n(x)) \in c_1) = 0$ or 1.

(5) For every sequence $x'_n \in E'$, $\mu(x; (x'_n(x)) \in c_0) = 0$ or 1.

(6) There exist no sequence $x'_n \in E'$ such that $\mu(x; (x'_n(x)) \in c_0) > 0$ and that $\mu(x; (x'_n(x) \notin l_\infty) > 0$.

(7) For every sequence $x'_n \in E'$, if there exists $(a_n) \in c_0$ such that $\mu(x; (a_n^{-1} x'_n(x)) \in l_\infty) > 0$, then $\mu(x; (x'_n(x)) \in c_0) = 1$.

(8) For every sequence $x'_n \in E'$, if there exists $(a_n) \in c_0$ such that $\mu(x; (a_n^{-1} x'_n(x)) \in l_\infty) > 0$, then $\mu(x; (x'_n(x) \in l_\infty) = 1$.

(9) For every sequence $x'_n \in E'$, if there exists $(a_n) \in c_0$ such that $\mu(x; (a_n^{-1} x'_n(x)) \in c_1) > 0$, then $\mu(x; (x'_n(x)) \in c_1) = 1$.

(10) For every sequence $x'_n \in E'$, if there exists $(a_n) \in c_0$ such that $\mu(x; (a_n^{-1} x'_n(x)) \in c_0) > 0$, then $\mu(x; (x'_n(x)) \in c_0) = 1$.

(11) For every sequence $x'_n \in E'$, if there exists $(a_n) \in c_0$ such that $\mu(x; (a_n^{-1} x'_n(x)) \in c_0) > 0$, then $\mu(x; (x'_n(x)) \in l_\infty) = 1$.

(12) For every sequence $x'_n \in E'$, $\mu(x; (x'_n(x)) \in l_p) = 0$ or 1 $(1 \leq p < \infty)$.

(13) For every closed convex balanced subset B, μ $(\cup_{n=1}^\infty nB) = 0$ or 1.

(14) For every lower semi-continuous semi-norm N(x) on E (admitting the value ∞), $\mu(x; N(x) < \infty) = 0$ or 1.

(15) For every compact convex balanced subset K, μ $(\cup_{n=1}^\infty nK) = 0$ or 1.

Main results: Theorem 1: The 0-1 laws (2)-(11) are equivalent. Theorem 2: Suppose that μ is a Radon probability measure. Then the 0-1 laws (2)- (11), (13) and (14) are all equivalent. Theorem 3: If μ is a convex Radon measure, then the 0-1 laws (2)-(11), (13), (14) and (15) are all equivalent.

Reviewer: D.Plachky

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60B05 Probability measures on topological spaces

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References:

[1] Borell, C., Convex measures on locally convex spaces, Ark. for Math., 12 (1974), 239-252. · Zbl 0297.60004 · doi:10.1007/BF02384761

[2] Dudley, R. M. and Kanter, M., Zero-one laws for stable measures, Proc. A. M.S., 45 (1974) , 245-252. · Zbl 0297.60007 · doi:10.2307/2040072

[3] Hoffmann-J0rgensen, J., Integrability of seminorms, the 0-1 law and affine kernel for product measures, Studia Math., 61 (1977), 137-159. · Zbl 0373.60014

[4] Krakowiak, W., Zero-one laws for stable and semi-stable measures on Banach spaces, Bull. Acad. Sci. Polonaise, 27 (1978) , 1045-1049. · Zbl 0407.60027

[5] Louie, D., Rajput, B. and Tortrat, A., A zero-one dichotomy theorem for r-semi- stable laws on infinite-dimensional linear spaces, Sankya Ser. A, 42 (1980) , 9-18. · Zbl 0487.60020

[6] Sato, H., Banach support of a probability measure in a locally convex space, Lecture Notes in Math., 526, Springer. · Zbl 0344.60005

[7] Takahashi, Y., Hilbertian support of probability measures on locally convex spaces and their applications, Hokkaido Math. /., 10 (1981), 57-74. · Zbl 0469.46032

[8] Zinn, J., Zero-one laws for non-Gaussian measures, Proc. A. M. S., 44 (1974) , 179-185. · Zbl 0309.60022 · doi:10.2307/2039252