Ding, Kequan

Note on an inequality for the integral $\int_a^b f(x)g(x)dx$. (Chinese) [Zbl 0606.26012]


The author has established the following Theorem 2. Let $f$ and $g \in L[a, b]$, $\int_a^b g(x)dx = 0$. Suppose that the function $G(x) = \int_a^x g(t)dt$ has finite zeros on $[a, b]$ only. If $G$ is quasi-convex or quasi-concave on every interval with end-points at adjoining zeros of $G$, then the inequality

$$|\int_a^b f(x)g(x)dx| \leq 2^{-1/q} (\|G\|, q) \text{Var}_{[a,b]}(f, p)$$

holds true, where $1 \leq p \leq \infty$, $1 \leq q \leq \infty$ and $1/p + 1/q = 1$. Furthermore, the constant $2^{-1/q}$ is best possible.

This result has improved a result of X. Wang.

Reviewer: Xianliang Shi

MSC:

26D15 Inequalities for sums, series and integrals
26A51 Convexity of real functions in one variable, generalizations
26A45 Functions of bounded variation, generalizations

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