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Relations d'orthogonalité sur les groupes de Mordell-Weil. (Relations of orthogonality on the Mordell-Weil groups). (French) [Zbl 0607.14014](#)

Théorie des nombres, Sémin. Paris 1984-85, Prog. Math. 63, 33-39 (1986).

[For the entire collection see [Zbl 0593.00007](#).]

Let E be an elliptic curve with complex multiplication by a quadratic imaginary field K , and assume that E is defined over K . Let \mathfrak{p} be a prime of K of *norm* p that is a prime of good reduction for E . For a number field F , galois over K with Galois group G , one defines the finite dimensional vector space V to be $E(F) \otimes \mathbb{Q}$, where $E(F)$ is the Mordell-Weil group of E . The vector space V is naturally a $K[G]$ -module equipped with a \mathbb{Q}_p -valued quadratic form h_p , the p -adic height function. Using the quadratic form h_p one can define a \mathbb{Q} -bilinear symmetric form $(\ , \)_p$ on $V \times V$ by $(P, Q)_p = (h_p(P+Q) - h_p(P) - h_p(Q))$. The main result of this paper gives a necessary and sufficient condition for two elements P and Q of V to be orthogonal with respect to $(\ , \)_p$. The relevant condition is the vanishing of a certain bilinear form on a submodule of $K[G] \times K[G]$ that is associated to the pair (P, Q) .

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MSC:

- [14G25](#) Global ground fields in algebraic geometry
- [14K22](#) Complex multiplication and abelian varieties
- [14H52](#) Elliptic curves
- [14H45](#) Special algebraic curves and curves of low genus

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elliptic curve with complex multiplication; Mordell-Weil group; p -adic height function