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Comparison of probability and eigenvalue methods for the Schrödinger equation. (English)

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Let D be a bounded domain in $\mathbb{R}^d$, $d \geq 1$ and a function q be bounded in D satisfying a Hölder condition on D. We say that ϕ is a solution of the Schrödinger boundary value problem (D,q,f) if $\phi \in C^{(2)}(D) \cap C^{(0)}(\bar{D})$, $(\Delta/2 + q)\phi = 0$ in D, and $\phi = f$ on ∂D. Such a solution is called "positive" if $\phi > 0$ in $\bar{D}$. If f is not specified, such a problem will be denoted by (D,q).

Denote by λ_1 the maximum eigenvalue of the operator $L = \Delta/2 + q$. Consider the following propositions: (i) ϕ (D,q,1,·) $\not\equiv \infty$ in D; (ii) there exists a positive solution of (D,q); (iii) $\lambda_1 < 0$.

The main result of the paper is the following Theorem: The three propositions (i), (ii), and (iii) are equivalent. Comparison of probability and eigenvalue methods for the Schrödinger boundary value problem has been carried out.

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MSC:

60H25	Random operators and equations (aspects of stochastic analysis)
35J10	Schrödinger operator, Schrödinger equation
35J25	Boundary value problems for second-order elliptic equations

Keywords:
Feynman-Kac functional; Schrödinger boundary value problem; maximum eigenvalue