Shustin, Eugenii

Tropical and algebraic curves with multiple points. (English) Zbl 1291.14084

Itenberg, Ilia (ed.) et al., Perspectives in analysis, geometry, and topology. On the occasion of the 60th birthday of Oleg Viro. Based on the Marcus Wallenberg symposium on perspectives in analysis, geometry, and topology, Stockholm, Sweden, May 19–25, 2008. Basel: Birkhäuser (ISBN 978-0-8176-8276-7/hbk; 978-0-8176-8277-4/ebook). Progress in Mathematics 296, 431-464 (2012).

Patchworking theorems are important steps in the proof of correspondence statements relating complex algebraic world and the tropical one. These theorems can be seen as variations of the patchworking construction proposed by O. Viro in the late 1970's.

The paper is devoted to a new version of a patchworking theorem. This version produces, out of appropriate plane tropical curves, complex (or real) algebraic curves of given genus which belong to a given linear system on a toric surface and have singularities with prescribed multiplicities at some fixed points. Though the theorem applies only to a specific class of plane tropical curves (it does not describe all tropical curves that can be obtained as tropical limits of complex curves under consideration), it is the key statement in several applications such as recursive Caporaso-Harris type formulas for Welschinger invariants of del Pezzo surfaces of degree ≥ 4. (Welschinger invariants can be seen as real analogs of genus zero Gromov-Witten invariants.)

For the entire collection see [Zbl 1230.00045].

Reviewer: Ilia Itenberg (Paris)

MSC:

14N35	Gromov-Witten invariants, quantum cohomology, Gopakumar-Vafa invariants, Donaldson-Thomas invariants (algebro-geometric aspects)
14N10	Enumerative problems (combinatorial problems) in algebraic geometry
14B07	Deformations of singularities
14P05	Real algebraic sets

Keywords:

patchworking; tropical curves; Welschinger invariants; del Pezzo surfaces

Full Text: DOI arXiv