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Periodic solutions of periodic competitive and cooperative systems. (English) Zbl 0609.34048

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Many mathematical models in the biological sciences give rise to the system $x_i' = F_i(x,t)$, $1 \leq i \leq n$, $x = (x_1, x_2, ..., x_n)$. The system is said to be cooperative or competitive according as $\partial F_i/x_j \geq$ or $\leq 0 (i \neq j)$. The periodic solutions, their basins of attraction and invariant manifolds are investigated for the case in which $F_i(x,t)$ is 2π-periodic in t. Competitive and cooperative mappings are introduced which possess the essential features of the Poincaré map of the system. The geometrical properties of these mappings and the discrete dynamical system they generate are investigated. The main techniques used are the Perron-Frobenius theory of positive matrices and invariant manifold theory. A complete description of the "phase portrait" of the discrete dynamical system generated by an orientation preserving planar cooperative map is obtained.

Reviewer: P.Smith

MSC:

34C25 Periodic solutions to ordinary differential equations

92D25 Population dynamics (general)

Keywords:

mathematical models; basins of attraction; Perron-Frobenius theory of positive matrices; invariant manifold theory

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