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Liouville-type theorems and decay estimates for solutions to higher order elliptic equations.

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Summary: Liouville-type theorems are powerful tools in partial differential equations. Boundedness assumptions of solutions are often imposed in deriving such Liouville-type theorems. In this paper, we establish some Liouville-type theorems without the boundedness assumption of nonnegative solutions to certain classes of elliptic equations and systems. Using a rescaling technique and doubling lemma developed recently in [*P. Poláčik* et al., Duke Math. J. 139, No. 3, 555–579 (2007; Zbl 1146.35038)], we improve several Liouville-type theorems in higher order elliptic equations, some semilinear equations and elliptic systems. More specifically, we remove the boundedness assumption of the solutions which is required in the proofs of the corresponding Liouville-type theorems in the recent literature. Moreover, we also investigate the singularity and decay estimates of higher order elliptic equations.

MSC:

35B53 Liouville theorems and Phragmén-Lindelöf theorems in context of PDEs
35J40 Boundary value problems for higher-order elliptic equations
35J47 Second-order elliptic systems
35B45 A priori estimates in context of PDEs

Keywords:

polyharmonic operators on half spaces; Dirichlet problem; Navier boundary condition; doubling property; without boundedness assumptions; rescaling technique

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