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Intervals of near-degeneracy in rational approximation. (English) [Zbl 0612.41023]

In this note it is shown that under certain circumstances nearly degenerate best rational approximations may exhibit unacceptable behavior in their denominators. Specifically, it is shown that if the best rational approximation from \( R^\ell_m[\alpha, \beta] \) to a given function \( f \) on \([\alpha, \beta] \) has an error curve with fewer than \( \ell + m + 1 \) zeros and there exist sequences \( \alpha_k \to \alpha \) and \( \beta_k \to \beta \) such that the best rational approximations \( r_k = p_k/q_k \) to \( f \) on \([\alpha_k, \beta_k] \) from \( R^\ell_n[\alpha_k, \beta_k] \) are non-degenerate then \( \inf\{q_k(x) : \alpha \leq x \leq \beta\} \to 0 \) as \( k \to \infty \) where a standard denominator normalization is required of \( R^\ell_m[\alpha_k, \beta_k] \) for all \( k \).

Reviewer: G.D. Taylor

MSC:
41A20 Approximation by rational functions
41A50 Best approximation, Chebyshev systems

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best rational approximations