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Metric fixed point theory on hyperconvex spaces: recent progress. (English) Zbl 1273.54052
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A metric space M is said to be hyperconvex if given any family $\{x_\alpha\}$ of points of M and any family $\{r_\alpha\}$ of real numbers satisfying $d(x_\alpha, x_\beta) \leq r_\alpha + r_\beta$, then $\bigcap_\alpha B(x_\alpha, r_\alpha) \neq \emptyset$. In this survey paper, the authors review the development of metric fixed point theory on hyperconvex metric spaces. In Section 4, they discuss the problem of characterizing proximal nonexpansive retracts of hyperconvex spaces and its connections to several problems in best approximation and best proximity pairs. In Section 5, recent developments on \mathbb{R} trees and metric fixed point theory are treated. In the last section, some recent advances on the theory of extensions of Hölder maps and their relation to extensions of uniformly continuous mappings under χ_0 hyperconvex conditions are presented. Some new results on the extension of compact mappings are also given.

Reviewer: **D. S. Diwan (Bhilai)**

MSC:

- 54H25** Fixed-point and coincidence theorems (topological aspects)
- 54E40** Special maps on metric spaces
- 54-02** Research exposition (monographs, survey articles) pertaining to general topology

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Keywords:

hyperconvex metric spaces; proximal nonexpansive retracts; best approximation; best proximity pairs

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