Hill, Michael A.

The equivariant slice filtration: a primer. (English) Zbl 1403.55003

Homology Homotopy Appl. 14, No. 2, 143-166 (2012).

The slice filtration in equivariant stable homotopy theory is constructed similarly to the Postnikov filtration, but using different kinds of test spheres. While the Postnikov tower commutes with ordinary suspensions, the slice tower commutes with suspensions by regular representations. Slice filtrations in general were first introduced by *V. Voevodsky* in the context of his work on the Milnor conjecture [in: Motives, polylogarithms and Hodge theory. Part I: Motives and polylogarithms. Papers from the International Press conference, Irvine, CA, USA, June 1998. Somerville, MA: International Press. 3–34 (2002; Zbl 1047.14012)]. The idea was adapted by *D. Dugger* into equivariant stable homotopy theory [*K*-Theory 35, No. 3–4, 213–256 (2005; Zbl 1109.14024)], and much further developed by the present author together with Hopkins and Ravenel in their work on the Kervaire-invariant-one problem; see [*M. A. Hill* et al., in: Current developments in mathematics, 2010. Somerville, MA: International Press. 1–43 (2011; Zbl 1249.55005) and in: Proceedings of the 17th Gökova geometry-topology conference, Gökova, Turkey, May 31 – June 4, 2010. Somerville, MA: International Press; Gökova: Gökova Geometry-Topology Conferences. 21–63 (2011; Zbl 1246.55014)] for the time being until the definitive account is published.

The present paper is written as an introduction to the slice filtration. Spectral sequences are mentioned only marginally, so that the author can focus on both the underlying geometric side (fixed points) and the algebraic aspects (Mackey functors) of the subject at hand. Despite is expository style, many results and examples in this paper are new, such as the slice filtration in the case of Eilenberg-Mac Lane spectra for Mackey functors of cyclic p-groups. This is not unlikely to become relevant in the author's work, with a different set of co-authors, on cyclic homology. The author also puts quite a bit of work into establishing a relation between the slice filtration and the Postnikov filtration: he proves that the slice tower is a re-indexed form of the Postnikov tower for the class of geometric spectra, for which the fixed and geometric fixed point spectra agree. This leads him to an interpretation of the slice tower for a general spectrum as an 'aggregation of stretched-out Postnikov sections, scaled by the order of the appropriate subgroups'. The final section contains a few conjectures. It should be noted, as the author did, that *J. Ullman* [Algebr. Geom. Topol. 13, No. 3, 1743–1755 (2013; Zbl 1271.55015)] has already proved some of the algebraic ones since the submission of this text.

An introduction is not a survey, and there are many other aspects of slice filtrations that the interested reader will want to learn and that are not even touched upon in the present text, thereby keeping its length reasonable. The author lets us in on his thinking and shares the points of view that he has developed during his work on the slice filtration. This text provides a very good start for learning the theory, in particular, due to its wealth of examples.

Reviewer: Markus Szymik (MR3007090)

MSC:

55N91 Equivariant homology and cohomology in algebraic topology
55P91 Equivariant homotopy theory in algebraic topology
55P92 Relations between equivariant and nonequivariant homotopy theory in algebraic topology

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