Bougerol, Philippe

Tightness of products of random matrices and stability of linear stochastic systems. (English)

Zbl 0614.60008
Ann. Probab. 15, 40-74 (1987).

The paper provides conditions implying tightness of sequences of n-fold convolutions μ^n, $n = 1, 2, ...$ of probability distributions μ on the set of real $d \times d$-matrices. These yield some applications to linear stochastic differential equations

$$(1) \quad dx_t = S_0 x_t dt + \sum_{i=1}^{r} S_i x_t \circ db_t^i$$

where $S_0, ..., S_r$ are fixed $d \times d$-matrices and b_t^i, $i = 1, ..., r$ are independent Brownian motions. Namely, if the largest Lyapunov exponent of the above system of equations is zero then its zero solution is stable in probability if and only if there exists an invertible matrix Q such that for $i = 0, ..., r$

$$QS_iQ^{-1} = \begin{pmatrix} A_i & & 0 \\ & K_i & \\ 0 & & B_i \end{pmatrix}$$

where K_i are skew-symmetric matrices and the largest Lyapunov exponents of systems of equations with the coefficients $A_0, ..., A_r$ and $B_0, ..., B_r$ in place of $S_0, ..., S_r$ in (1) are strictly negative.

Reviewer: Y.Kifer

MSC:

60B15 Probability measures on groups or semigroups, Fourier transforms, factorization
60B10 Convergence of probability measures
93E15 Stochastic stability in control theory
60H10 Stochastic ordinary differential equations (aspects of stochastic analysis)
60H25 Random operators and equations (aspects of stochastic analysis)

Keywords:

products of random matrices; stability of linear stochastic systems; tightness of sequences of n-fold convolutions; largest Lyapunov exponent

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