Smith, Peter
An introduction to Gödel's theorems. 2nd ed. (English) Zbl 1270.03002
Cambridge Introductions to Philosophy. Cambridge: Cambridge University Press (ISBN 978-1-107-60675-3/pbk; 978-1-107-02284-3/hbk). xvi, 388 p. (2013).

This is the second edition that was anticipated (and welcomed) in the review of its first edition Zbl 1154.03002. Fortunately, (almost) all the points in that review have been considered in the new edition; only the main criticism of the review "the lack of a full proof for the second theorem" still remains, but the author apologetically does "say rather more than before" and provides excellent references for interested readers to study "a full-blown, warts-and-all proof of the Second Theorem". Another deficiency of the first edition on proving "that the Ackermann function is recursive but not p.r." has been repaired in the new edition. The theorem of V. H. Dyson, spelled out in the review of the first edition, has been added as Theorem 16.4 in the second edition, and the status of the second theorem for weak theories like $\mathbf{Q}$ or $\mathbf{I}\Delta_0$ has been rewritten with much more care. This all shows the influence of the review of the first edition on the second.

As for the second edition, it "is over twenty pages longer, but that isn't because there is much new material". Rather, the author has "mostly used the extra pages to make the original book more reader-friendly; there has been a lot of rewriting and rearrangement, particularly in the opening chapters. Perhaps the single biggest change is in using a more traditional line of proof for the adequacy of Robinson Arithmetic ($\mathbf{Q}$) for capturing all the primitive recursive functions." And, indeed, the new edition is a beautiful and mature revision of the first. For example, the fantastic proof of Bézout's lemma on page 116 (used in Gödel's beta function), the proof of p.r. adequacy of $\mathbf{Q}$ in Chapter 17, the theorem of speed-up in Chapter 28, a nice introduction to and a very nice treatment of second-order arithmetic in Chapter 29, mentioning Jeroslow's lemma regarding Gödel' second incompleteness theorem in Chapter 33, a very neat and modern introduction to Turing machines in Chapter 41 and an interesting discussion on the Church's thesis (and possible ways of proving it) in Chapter 45 are all the strong and excellent points of the new (and some of them already in the first) edition of the book.

The website `http://www.logicmatters.net/igt/`, which is recommended to consult before reading the book, contains supplementary material on and about the book, with a list of "a handful of minor corrections" at `http://www.logicmatters.net/resources/pdfs/godelbook/Corrections2.pdf`, in which the following misprints are missing:

(1) in line 2 of page 161 (Chapter 22) Δ_0 should be $\mathbf{I}\Delta_0$;

(2) in line 1 of page 245 (Chapter 33) "In the Chapter" should be "In Chapter";

(3) in lines 1, 6 and 7 of page 307 (Chapter 40) all the Rjn's should be "$R(j,n)$";

(4) in line 17 of page 363 (Chapter 45) "we already implied by" should be "we already suggested by".

One thing that many have wondered about is the philosophy (if any) of naming the two most great achievements of Gödel as "completeness" and "incompleteness"; and what is the relation of these two theorems (if any)? The author emphasizes that the incompleteness theorem is better read as "incompletability", and the current reviewer agrees with this. But that does not answer the historical question of naming these theorems. Well, the completeness theorem states that *the first-order logic is (strongly) complete with respect to the semantics of structures*, in the sense that *a theory proves a sentence if and only if the sentence is true in all the models of that theory*. In particular, an arithmetical sentence φ is provable in Peano Arithmetic $\mathbf{PA}$, that is $\mathbf{PA}\vdash\varphi$, if and only if φ is true in all the models of $\mathbf{PA}$, that is $\mathbf{PA}\models\varphi$. By Gödel's incompleteness theorem $\mathbf{PA}$ is a proper subset of the set of all true first-order arithmetical sentences $\mathcal{T}_A$. This is prettily written on page 217 as $\{\mathbf{PA},\vdash\}=\{\mathbf{PA},\models\}\subset\mathcal{T}_A$. One reason that the other theorem of Gödel is called "incompleteness" is the incompleteness of the second-order arithmetic (which is implied by this theorem). This, also, is written prettily on page 217 as $\{\mathbf{PA}_2,\vdash\}\subset\{\mathbf{PA}_2,\models\}=\mathcal{T}_{2A}$. So, while the first-order logic is complete (with respect to its semantics) the second-order logic is not complete (one reason comes from the first-order incompleteness of arithmetic by Gödel's theorem); hence the names. Though, the author's discussion on page 217 (Chapter 29) is fantastic, the reviewer wishes that there was more elaboration on the names of these two great theorems of Gödel. But all of these are implicit in Chapter 29.

Summing up, this is one of the very best books on the subject that has ever been written, and of course the most modern of all the existing ones so far. I have used the first edition in a course and (along with the students) learned a lot; and now I am using the second edition for my courses, and it is almost a fixed reference cited in my (and my postgraduate students') research papers. I specially love this second edition and recommend it for all the interested scholars in Gödel's most well-known theorems that are rightfully regarded as one of the greatest achievements of human knowledge in the twentieth century.

Reviewer: Saeed Salehi (Tabriz)

MSC:

03-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to mathematical logic and foundations
03-02 Research exposition (monographs, survey articles) pertaining to mathematical logic and foundations
03-03 History of mathematical logic and foundations
03F30 First-order arithmetic and fragments
03F40 Gödel numberings and issues of incompleteness

Keywords:

Godel's incompleteness theorems

Full Text: DOI