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Approximations by the Cauchy-type integrals with piecewise linear densities. (English)
Zbl 1274.30146

Summary: The paper is a contribution to the complex variable boundary element method, shortly CVBEM. It is focused on Jordan regions having piecewise regular boundaries without cusps. Dini continuous densities whose modulus of continuity $\omega(\cdot)$ satisfies

$$\limsup_{s \downarrow 0} \omega(s) \ln \frac{1}{s} = 0$$

are considered on these boundaries. Functions satisfying the Hölder condition of order α, $0 < \alpha \leq 1$, belong to them. The statement that any Cauchy-type integral with such a density can be uniformly approximated by a Cauchy-type integral whose density is a piecewise linear interpolant of the original one is proved under the assumption that the mesh of the interpolation nodes is sufficiently fine and uniform. This result ensures the existence of approximate CVBEM solutions of some planar boundary value problems, especially of the Dirichlet ones.

MSC:

30E10	Approximation in the complex plane
30E20	Integration, integrals of Cauchy type, integral representations of analytic functions in the complex plane
65N12	Stability and convergence of numerical methods for boundary value problems involving PDEs
65N38	Boundary element methods for boundary value problems involving PDEs

Keywords:

Cauchy-type integral; Dini continuous density; piecewise linear interpolation; uniform convergence; complex variable boundary element method

Full Text: DOI

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