Loi, N. V.

On the radical theory of involution algebras. (English) Zbl 0615.16008

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In the variety of involution K-algebras over a commutative ring K with 1, a subclass $\mathfrak{S}$ is the semisimple class of a Kurosh-Amitsur radical, if $\mathfrak{S}$ is regular, coinductive, closed under extensions and the class $\mathcal{SUS}$ is hereditary where $\mathcal{S}$ and $\mathcal{U}$ denote the semisimple and upper radical operator, respectively. Let $\mathfrak{C}$ be a subclass of involution algebras satisfying condition (ID): $A^{id} \in \mathfrak{C}$ if and only if $A^{-id} \in \mathfrak{C}$ whenever $A^2 = 0$; if $\mathfrak{C}$ is regular, then $\mathcal{U}\mathfrak{C}$ satisfies A-D-S, and if $\mathfrak{C}$ is homomorphically closed, then its lower radical $\mathcal{L}\mathfrak{C}$ satisfies A-D-S (which means that the radical of any involution ideal is an involution ideal in the involution algebra). Let $\mathfrak{F}$ be a regular, coinductive subclass which is closed under extensions. Then the following are equivalent: i) $\mathcal{U}\mathfrak{F}$ satisfies A-D-S, ii) $\mathfrak{F}$ satisfies (ID), iii) if $A^* \in \mathfrak{F}$ and $A^2 = 0$, then $A^\circ \in \mathfrak{F}$ for every involution $\circ$, iv) if $A^* \in \mathfrak{F}$ and $A^2 = 0$, then $A^{-*} \in \mathfrak{F}$, v) if $A^* \in \mathfrak{F}$ and A is nilpotent, then every nilpotent involution algebra built on A is in $\mathfrak{F}$. An example is given showing that a coradical class need not be a semisimple class.

Reviewer: R.Wiegandt

MSC:

16W10 Rings with involution; Lie, Jordan and other nonassociative structures
16Nxx Radicals and radical properties of associative rings

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semisimple class; Kurosh-Amitsur radical; upper radical; involution algebras; lower radical; involution ideal; A-D-S; coradical class

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