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**Double coverings of curves and non-Weierstrass semigroups.** (English) Zbl 1270.14014

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Let $X$ be a (non-singular, projective, irreducible, algebraic) curve defined over an algebraically closed field of characteristic zero, and let $P \in X$. The Weierstrass semigroup $H(P)$ of $X$ at $P$ is the set of poles of regular function on $X \setminus \{P\}$. Thus $H(P)$ is indeed a subsemigroup of the additive semigroup $\mathbb{N}_0$ such that $\#(\mathbb{N}_0 \setminus H(P)$ equals the genus of $X$ (The Weierstrass gap theorem).

Let $H$ be a subsemigroup of $(\mathbb{N}_0, +)$ which is named numerical provided that $G(H) := \mathbb{N}_0 \setminus H)$ is finite; the genus of $H$ is $\#G(H)$. The subject matter addressed in the paper under review is related to the following question posed by Hurwitz around 1892 [*A. Hurwitz*, Math. Ann. XLI, 403–442 (1893; JFM 24.0380.02)]: Is any numerical semigroup $H$ equal to the Weierstrass semigroup at some point of a curve? If this is so, $H$ is called Weierstrass. The answer to this question is in general negative as *R.-O. Buchweitz* pointed out around 1980 [Lect. Notes Math. 777, 205–220 (1980; Zbl 0428.32016)]. He observed the following by considering elements of $G(H)$. For an integer $m \geq 2$, let $G_m(H)$ be the set of all sums of $m$ elements of $G(H)$. Thus if $H$ of genus $g$ were Weierstrass, then $\#G_m(H) \leq (2m-1)(g-1)$ $(*)$, the dimension of a $m$-pluricanonical divisor of a curve of genus $g$. In fact, Buchweitz constructed non-Weierstrass semigroups by contradicting condition $(*)$; see also [*J. Komeda*, Semigroup Forum 57, No. 2, 157–185 (1998; Zbl 0922.14022)]. The least genus of Buchweitz's examples is $g = 16$. Numerical semigroups of genus at most eight are always Weierstrass; see [*J. Komeda* and *A. Ohbuchi*, Bull. Braz. Math. Soc. (N.S.) 39, No. 1, 109–121 (2008; Zbl 1133.14307)] and the references therein. Let $\ell_g = \ell_g(H)$ be the biggest element of $G(H)$. By the semigroup property, $\ell_g \leq 2g - 1$. If $\ell_g \geq 2g - 4$, then condition $(*)$ is always true for any numerical semigroup; cf. [*G. Oliveira*, Semigroup Forum 69, No. 3, 423–430 (2004; Zbl 1076.20052)] and the references therein. However, non-Weierstrass numerical semigroups with $\ell_g \geq 2g-4$ do exist: *G. Oliveira* and *K.-O. Stöhr* [Geom. Dedicata 67, No. 1, 45–63 (1997; Zbl 0904.14018)], *F. Torres* [Commun. Algebra 23, No. 11, 4211–4228 (1995; Zbl 0842.14023)]; see also *N. Medeiros* [J. Pure Appl. Algebra 170, No. 2–3, 267–285 (2002; Zbl 1039.14015)]. The basic tool in constructing such non-Weierstrass semigroups is the use of certain covering of curves and Buchweitz's examples as a building block (Stöhr).

On the other hand, let $m(H)$ be the first positive element of a numerical semigroup $H$. If $m(H) \leq 5$, $H$ is always Weierstrass; see [*J. Komeda*, Manuscripta. Math. 76, No. 2, 193–211 (1992; Zbl 0770.30038)] and the references therein. There exist non-Weierstrass semigroups $H$ whenever $m(H) \geq 13$ (e.g. Buchweitz, loc. cit.). In the article under review, the author constructs examples of non-Weierstrass semigroups $H$ with $m(H) = 8$ and with $m(H) = 12$. To explain his method, for a numerical semigroup $\tilde{H}$ let us consider the associated numerical semigroup $d_2(\tilde{H}) := \{h/2 : h$ is even

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14H55 Riemann surfaces; Weierstrass points; gap sequences
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