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Summary: The migration of melt through the mantle of the Earth is governed by a third-order nonlinear partial differential equation for the voidage or volume fraction of melt. The partial differential equation depends on the permeability of the medium which is assumed to be a function of the voidage. It is shown that the partial differential equation admits, as well as translations in time and space, other Lie point symmetries provided the permeability is either a power law or an exponential law of the voidage or is a constant. A rarefactive solitary wave solution of the partial differential equation is derived in the form of a quadrature for the exponential law for the permeability.

MSC:

86A99 Geophysics
74J35 Solitary waves in solid mechanics
35Q86 PDEs in connection with geophysics

Full Text: DOI

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