Meschenmoser, D.; Shashkin, A.

Functional central limit theorem for the measures of level surfaces of the Gaussian random field. (English. Russian original) Zbl 1278.60052

Theory Probab. Appl. 57, No. 1, 162-172 (2013); translation from Teor. Veroyatn. Primen. 57, No. 1, 168-178 (2012).

Let $X = \{X_s, s \in \mathbb{R}^d\}$ be a centered stationary and isotropic Gaussian random field having C^1 realizations. For $t > 0$ and $x \in \mathbb{R}$ introduce the stochastic process $N_t(x) = t^{-d/2}(\mathcal{H}_{d-1}(B_t(x)) - \mathrm{E}\mathcal{H}_{d-1}(B_t(x)))$ where $B_t(x) = \{s \in [0,t]^d : X_s = x\}$ and $\mathcal{H}_{d-1}$ is the $(d-1)$-dimensional Hausdorff measure of a set in $\mathbb{R}^d$. The authors prove that if the covariance function R of a field X satisfies certain conditions (involving first and second derivatives of R) then the family of random elements $\{N_t, t > 0\}$ converges in distribution as $t \to \infty$ in the space $L^2(\mathbb{R}, \mu)$ to the specified centered Gaussian random element. Here μ denotes the standard Gaussian measure on $\mathbb{R}$. Thus the Hilbert space functional central limit theorem is established for the Hausdorff measures of the level sets of a field X.

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60G60 Random fields

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